Problem Set #4 - due Friday, September 16

1. Let $\mathbb{R}^\infty$ denote the direct product of countably many copies of $\mathbb{R}$ (i.e. the set of all real-valued sequences), and let P and B denote the topological spaces obtained by giving $\mathbb{R}^\infty$ the product and box topologies, respectively.

(a) Let $f : X \to \mathbb{R}^\infty$ be a function from a topological space X to the product $\mathbb{R}^\infty$. Show that if f is continuous when regarded as a function from X to B then it is also continuous when regarded as a function from X to P.

(b) Give an example of a function $f : \mathbb{R} \to \mathbb{R}^\infty$ that is continuous as a function from $\mathbb{R}$ to P but is not continuous as a function from $\mathbb{R}$ to B.

2. Consider the six topologies on the set of real numbers: $\mathbb{R}$, $\mathbb{R}_d$, $\mathbb{R}_t$, $\mathbb{R}_{fc}$, $\mathbb{R}_{cc}$, $\mathbb{R}_l$. Which of these has a countable basis?

3. Let A be a subspace of the topological space X.

(a) Show that if $f : X \to Y$ is continuous then $f|_A : A \to Y$ is also continuous, where $f|_A$ denotes the restriction of f to A.

(b) Give an example of a continuous function $f : (0, 1) \to \mathbb{R}$ which is not the restriction of any continuous function from $[0, 1]$ to $\mathbb{R}$.

4. Let $\mathcal{T}$ be the smallest topology on the set of real numbers with the property that every subset of the rational numbers is open. Let X denote the space consisting of the set of real numbers with the topology $\mathcal{T}$.

(a) Is $\mathcal{T}$ the discrete topology?

(b) Find all continuous functions from X to $\mathbb{R}$, where $\mathbb{R}$ denotes the usual topology on the real numbers.

5. Consider a line in the plane, with the topology that it inherits as a subspace of the product space $\mathbb{R} \times \mathbb{R}_l$. Describe (up to homeomorphism) all of the different topological spaces that occur as subspaces of this type.