Math 445 – Fall 2011 Name: ______________________________

**Problem Set #5 - due Monday, September 26**

**1.** Prove that the dictionary order topology on $\mathbb{R} \times \mathbb{R}$ is the same as the product topology on $\mathbb{R}_d \times \mathbb{R}$.

**2.** Let $X$ be a linearly ordered set with the order topology. For $x, y \in X$, if $x < y$ and $(x, y) = \emptyset$ then we will say that $x$ is the *predecessor* of $y$ and $y$ is the *successor* of $x$. Prove that $\overline{(a, b)} = [a, b]$ if and only if $a$ has no successor and $b$ has no predecessor.

**3.** Let $X$ and $Y$ be topological spaces and let $x_0 \in X$. Prove that the function $f : Y \to X \times Y$ given by $f(y) = (x_0, y)$ is a homeomorphism from $Y$ to the subspace $f(Y)$.

**4.** Let $Y$ be a linearly ordered set with the order topology and let $f : X \to Y$ and $g : X \to Y$ be continuous.

(a) Prove that $\{x \mid f(x) \leq g(x)\}$ is a closed subset of $X$.

(b) Prove that the function $h : X \to Y$ given by $h(x) = \min\{f(x), g(x)\}$ is continuous

**5.** Let $P$ and $B$ denote the topological spaces obtained by giving the countable product of copies of $\mathbb{R}$ the product and box topologies, respectively. Let $S$ be the subset of this product consisting of all points for which all but finitely many of the coordinates are 0. Find the closure of $S$ in $P$ and in $B$.