Math 445 – Fall 2011 Name: ____________________

Problem Set #7 - due Monday, October 17

1. Suppose X and Y are (non-empty) topological spaces and $f : X \to Y$ is a continuous map which is both open and closed. Show that if Y is connected then f is surjective.

2. Let $f : X \to Y$ be a continuous map from a topological space X to a topological space Y. Suppose that each point $x \in X$ has a neighborhood U such that the restriction $f|_U : U \to Y$ is a constant function. Show that if X is connected then f is a constant function.

3. Let X be the subset of $[0, 1] \times [0, 1]$ consisting of points (x, y) such that either x and y are both rational or x and y are both irrational. Is X connected? Is it locally connected? Is it path-connected?

4. Give an example of a connected space X which is not locally connected at any point, i.e. no point of X has a neighborhood basis consisting of connected sets.

5. Prove that a space X is locally connected if and only if all of the connected components of any open subset of X are open.