Math 121 – Exam 1 Solutions

1. (20 pts) The graph of a polynomial function $f(x)$ is shown below. Answer the following:

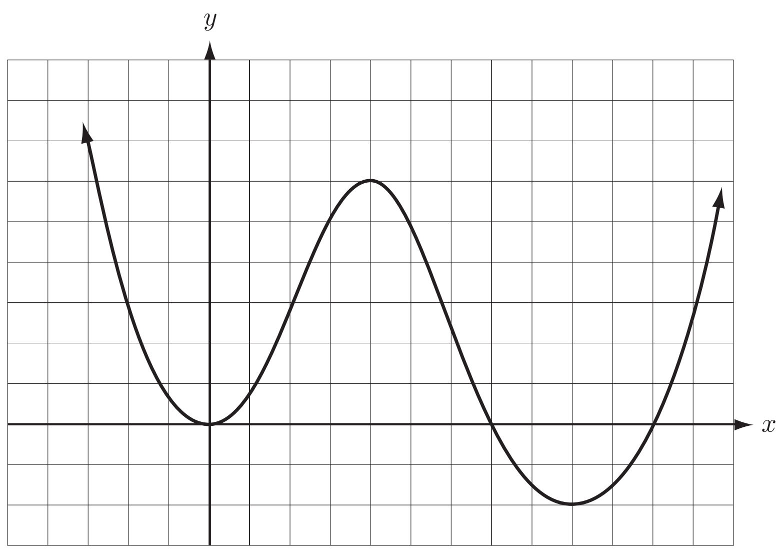
   (a) Is $f(x)$ an even function, an odd function, or neither?

   (b) On what interval(s) is $f(x)$ increasing?

   (c) What is the minimum degree of $f(x)$?

   (d) Locate all real zeros of $f(x)$ and state whether the multiplicity of each zero is even or odd.

(Each square in the grid has a side of length 1.)



Solution:

(a) The function is neither odd nor even.

(b) $f(x)$ is increasing on $(0, 4)$ and $(9, \infty)$

(c) The minimum degree of $f(x)$ is 4 (there are 3 turning points).

(d) The real zeros of $f(x)$ are $x = 0$ (even multiplicity), $x = 7$ (odd multiplicity), and $x = 11$ (odd multiplicity).

2. (15 pts) Write the rule of the function $g(x)$ obtained by performing the following transformations on the function $f(x) = 2x^2 + 1$: (1) shift 4 units downward, (2) shift 1 unit to the right, and (3) expand vertically by a factor of 5.

Solution:

$$2x^2 + 1 \quad \rightarrow \quad 2x^2 - 3 \quad \text{(after shifting 4 units downward)}$$
$$2x^2 - 3 \quad \rightarrow \quad 2(x-1)^2 - 3 \quad \text{(after shifting 1 unit to the right)}$$
$$2(x-1)^2 - 3 \quad \rightarrow \quad 10(x-1)^2 - 15 \quad \text{(after expanding vertically by a factor of 5)}$$

Thus, $\boxed{g(x) = 10(x-1)^2 - 15}$.

3. (10 pts) Find the vertex and axis of symmetry of $f(x) = 3x^2 - 24x - 17$.

Solution: To find the vertex and axis of symmetry, we will complete the square:

$$\begin{aligned} f(x) &= 3x^2 - 24x - 17 \\ &= 3(x^2 - 8x) - 17 \\ &= 3(x^2 - 8x + 16) - 17 - 3(16) \\ &= 3(x-4)^2 - 65 \end{aligned}$$

Therefore, the vertex is $\boxed{(4, -65)}$ and the axis of symmetry is $\boxed{x = 4}$.

4. (10 pts) Solve the inequality: $\dfrac{x+2}{x-2} \geq 0$.

Solution: To solve the inequality, we first note that the roots of the numerator and denominator are $x = -2$ and $x = 2$, respectively. Let $f(x) = \frac{x+2}{x-2}$. Then, using the following table:

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Number Chosen	-3	0	3
Value of f	$f(-3) = \frac{1}{5}$	$f(0) = -1$	$f(3) = 5$
Conclusion	positive	negative	positive

Since $f(x) \geq 0$, the solution is $\boxed{x \leq -2 \text{ or } x > 2}$.

5. (20 pts) Consider the rational function $R(x) = \dfrac{x^2(x+2)}{x^2 + 4x + 4}$.

   (a) What is the domain of $R(x)$?

   (b) Find all vertical asymptotes of $R(x)$, if any.

   (c) Does the graph of $R(x)$ contain a hole? If so, where?

   (d) Find the oblique asymptote of $R(x)$.

6. (15 pts) Consider the function $f(x) = 2x^3 + x^2 + 4x - 15$.

   (a) List all possible rational zeros of $f(x)$.

   (b) Given that $x = -1 + 2i$ is a zero of $f(x)$, find all remaining zeros.

Solution:

(a) The factors of $a_0 = -15$ are $\pm 1, \pm 3, \pm 5, \pm 15$. The factors of $a_3 = 2$ are $\pm 1, \pm 2$. Therefore, the possible rational zeros are:

$$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$$

(b) Since $x = -1 + 2i$ is a zero, so is its conjugate $x = -1 - 2i$. Since $f\left(\frac{3}{2}\right) = 0$, the last remaining zero is $x = \frac{3}{2}$.

7. (10 pts) Let $f(x) = \frac{2}{x-3}$ and $g(x) = x + 1$.

   (a) Compute $(f \circ g)(1)$.

   (b) Write the rule for $f^{-1}(x)$.

Solution:

(a) $(f \circ g)(1) = f(g(1)) = f(2) = -2$

(b) To find the rule for $f^{-1}(x)$, we first write:

$$y = \frac{2}{x-3}$$

Switching x and y we have:

$$x = \frac{2}{y-3}$$

Solving for y we get:

$$\begin{aligned} x &= \frac{2}{y-3} \\ y - 3 &= \frac{2}{x} \\ y &= 3 + \frac{2}{x} \end{aligned}$$

Therefore, $\boxed{f^{-1}(x) = 3 + \frac{2}{x}}$.