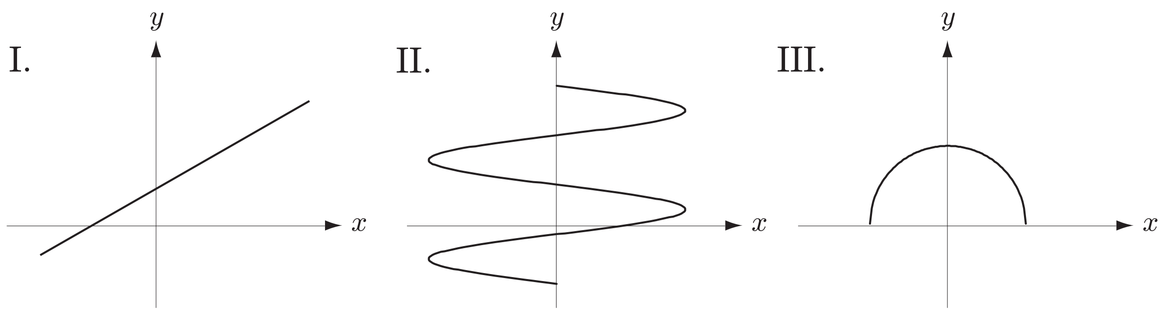
Math 210 – Exam 1 Solutions

1. (20 pts) (a) Find the solution to the equation:

$$\frac{\sqrt{x+6}}{x} = 1$$

(b) Which of the following graphs define y as a function of x?



Solution:

(a) The solution is $\boxed{x = 3}$. You can obtain this answer by either (1) guessing, (2) graphing the function $f(x) = \sqrt{x+6}x - 1$ on your calculator and finding the zero, or (3) using algebra:

$$\begin{aligned}
\frac{\sqrt{x+6}}{x} &= 1 \\
\sqrt{x+6} &= x \\
(\sqrt{x+6})^2 &= x^2 \\
x + 6 &= x^2 \\
x^2 - x - 6 &= 0 \\
(x-3)(x+2) &= 0 \\
\boxed{x = 3},\ x &= -2
\end{aligned}$$

However, $x = -2$ does not work. If we test it by plugging it back into the original equation we get:

$$\begin{aligned}
\frac{\sqrt{-2+6}}{-2} &= 1 \\
\frac{2}{-2} &= 1 \\
-1 &= 1
\end{aligned}$$

which is not true.

(b) Graphs $\boxed{\text{I and III}}$ define y as a function of x. Each vertical line drawn on the graph passes through the graph at most one time.

2. (15 pts) Find the domain of the function:

$$f(x) = \frac{x}{\sqrt{1-x^2}}$$

Solution: The expression inside the square root must be greater than zero. Note: It cannot be equal to zero since we cannot divide by zero:

$$\begin{aligned} 1 - x^2 &> 0 \\ 1 &> x^2 \\ x^2 &< 1 \\ \boxed{-1 < x < 1} \end{aligned}$$

3. (15 pts) Write the rule of the function $g(x)$ obtained by performing the following transformations on $f(x) = 2x^2$:

   (1) shift 4 units downward,

   (2) expand vertically by a factor of 2, and

   (3) shift 3 units to the right.

   Solution: $f(x)$ is transformed as follows:

$$f(x) = 2x^2 \;\rightarrow\; 2x^2 - 4 \;\rightarrow\; 2(2x^2 - 4) \;\rightarrow\; \boxed{2(2(x-3)^2 - 4) = g(x)}$$

4. (20 pts) (a) Let $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x} + x$. Find the composite function $(g \circ f)(x)$.

   (b) Find the inverse of the function $f(x) = \frac{1}{2x+1}$.

   Solution:

   (a) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}} + \frac{1}{x} = \boxed{\frac{1}{\sqrt{x}} + \frac{1}{x}}$

   (b) Start by writing the function as $y = \frac{1}{2x+1}$. Then interchange x and y to get:

$$x = \frac{1}{2y+1}$$

   Now solve for y:

$$\begin{aligned} x &= \frac{1}{2y+1} \\ x(2y+1) &= 1 \\ 2xy + x &= 1 \\ 2xy &= 1 - x \\ y &= \frac{1-x}{2x} \end{aligned}$$

   The inverse is then $\boxed{f^{-1}(x) = \frac{1-x}{2x}}$.

5. (15 pts) You and a friend are planning a trip to Cedar Point, which is about 300 miles east of Chicago. You leave at 8AM while your friend doesn't leave until 9AM. If you drive at an average of 60 miles/hour, how fast must your friend drive so that you arrive at Cedar Point at the same time? (hint: distance = speed $\times$ time)

Solution: If you drive 60 mph for 300 miles then it takes you:

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{300 \text{ miles}}{60 \text{ mph}} = 5 \text{ hours}$$

to get there. Therefore, you will get there at 1PM.

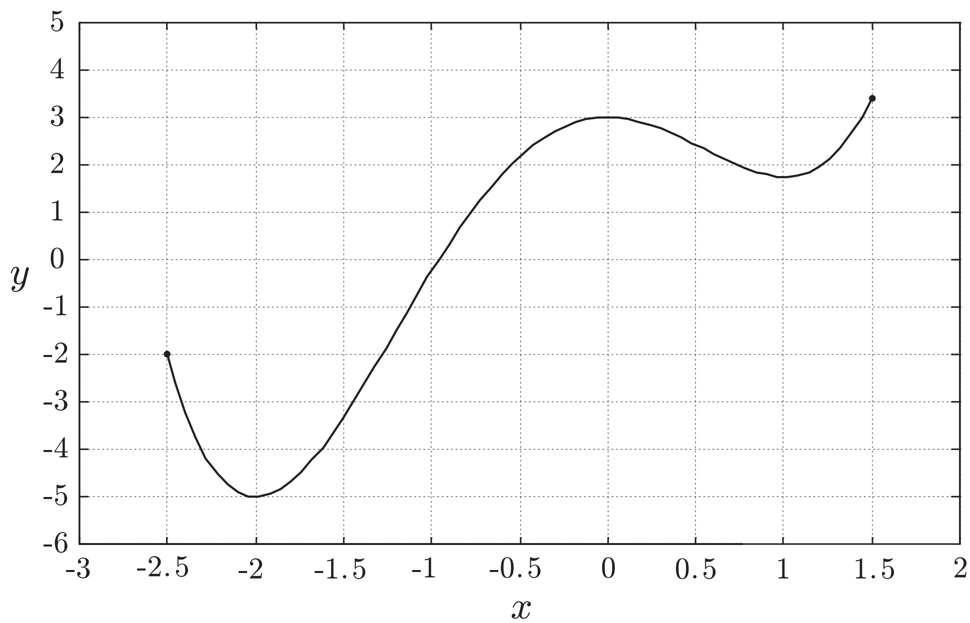
In order for your friend to arrive at the same time, it must take him/her 4 hours to get there since he/she leaves at 9AM. Driving 300 miles for 4 hours, your friend's average speed should be:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{300 \text{ miles}}{4 \text{ hours}} = \boxed{75 \text{ mph}}$$

6. (15 pts) For the function $f(x) = \frac{3}{4}x^4 + x^3 - 3x^2 + 3$ with $-2.5 \le x \le 1.5$ (plotted below), find:

   (a) all local maxima and minima,

   (b) the interval(s) where the function is increasing.



Solution:

(a) From the graph, there is a $\boxed{\text{local maximum at } (0, 3)}$ and a $\boxed{\text{local minimum at } (-2, -5)}$.

There is another local minimum with an x-coordinate of 1 but the y-coordinate is not obvious. We can obtain the y-coordinate by plugging $x = 1$ into the given equation:

$$f(1) = \frac{3}{4}(1)^4 + (1)^3 - 3(1)^2 + 3 = \frac{3}{4} + 1 - 3 + 3 = 1\frac{3}{4} = 1.75$$

Thus, the other $\boxed{\text{local minimum is at } (1, 1.75)}$.

(b) The function is $\boxed{\text{decreasing on the intervals } (-2.5, -2) \text{ and } (0, 1)}$.

The function is $\boxed{\text{increasing on the intervals } (-2, 0) \text{ and } (1, 1.5)}$.