Math 121 – Quiz 3 Solution

1. Construct a rational function $R(x)$ that has the following properties:
   - $x = 1$ and $x = -2$ are vertical asymptotes
   - $y = 3$ is a horizontal asymptote
   - the x-intercept is at $x = 0$
2. Solve the inequality $\dfrac{x-3}{x+1} > 0$.

Solution:

1. Since $x = 1$ and $x = -2$ are vertical asymptotes, we know that:

$$R(x) = \frac{p(x)}{(x-1)(x+2)}$$

   Since $y = 3$ is a horizontal asymptote, the degree of $p(x)$ is 2 and the coefficient of x^2 is 3. Also, since the x-intercept is at $x = 0$ we know that $p(0) = 0$. Therefore, we can say that $p(x) = 3x^2$. So, the function $R(x)$ is:

$$R(x) = \frac{3x^2}{(x-1)(x+2)}$$

2. Using the fact that the zeros of the numerator and denominator of $f(x) = \dfrac{x-3}{x+1}$ are $x = 3, -1$, we set up the following table:

Interval	$(-\infty, -1)$	$(-1, 3)$	$(3, \infty)$
Number Chosen	-2	0	4
Value of f	$f(-2) = 5$	$f(0) = -3$	$f(4) = \frac{1}{5}$
Location of graph	above x-axis	below x-axis	above x-axis

   Since $f(x) > 0$, the solution is $x < -1$ or $x > 3$.