Math 121 – Section 3.3 Solutions

11-18. (in order) C, E, F, A, G, B, H, D

20. $f(x) = 2x^2$ is the function x^2 vertically stretched by a factor of 2

36. Completing the square on $f(x) = x^2 - 4x$, we have:

$$\begin{aligned} f(x) &= x^2 - 4x \\ &= (x^2 - 4x + 4) - 4 \\ &= (x-2)^2 - 4 \end{aligned}$$

(a) the graph opens up; the vertex is $(2, -4)$; the axis of symmetry is $x = 2$; the y-intercept is $y = 0$; the x-intercepts are $x = 0$ and $x = 4$
(b) the domain is all real numbers; the range is $[-4, \infty)$
(c) the function is decreasing on the interval $(-\infty, 2)$; the function is increasing on the interval $(2, \infty)$

41. Completing the square on $f(x) = x^2 + 2x - 8$, we have:

$$\begin{aligned} f(x) &= x^2 + 2x - 8 \\ &= (x^2 + 2x + 1) - 8 + 1 \\ &= (x+1)^2 - 7 \end{aligned}$$

(a) the graph opens up; the vertex is $(-1, -7)$; the axis of symmetry is $x = -1$; the y-intercept is $y = -8$; the x-intercepts are $x = -1 \pm \sqrt{7}$
(b) the domain is all real numbers; the range is $[-7, \infty)$
(c) the function is decreasing on the interval $(-\infty, -1)$; the function is increasing on the interval $(-1, \infty)$

44. Completing the square on $f(x) = x^2 + 6x + 9$, we have:

$$\begin{aligned} f(x) &= x^2 + 6x + 9 \\ &= (x+3)^2 \end{aligned}$$

(a) the graph opens up; the vertex is $(-3, 0)$; the axis of symmetry is $x = -3$; the y-intercept is $y = 9$; the x-intercept is $x = -3$
(b) the domain is all real numbers; the range is $[0, \infty)$
(c) the function is decreasing on the interval $(-\infty, -3)$; the function is increasing on the interval $(-3, \infty)$

53. The vertex is at $(-1, -2)$. Therefore, we know that:

$$f(x) = a(x+1)^2 - 2$$

The point $(0, -1)$ is on the graph. Therefore,

$$\begin{aligned} f(0) = a(0+1)^2 - 2 &= -1 \\ a - 2 &= -1 \\ a &= 1 \end{aligned}$$

The quadratic function is $\boxed{f(x) = (x+1)^2 - 2}$.

54. The vertex is at $(2, 1)$. Therefore, we know that:

$$f(x) = a(x-2)^2 + 1$$

The point $(0, 5)$ is on the graph. Therefore,

$$\begin{aligned} f(0) = a(0-2)^2 + 1 &= 5 \\ 4a + 1 &= 5 \\ 4a &= 4 \\ a &= 1 \end{aligned}$$

The quadratic function is $\boxed{f(x) = (x-2)^2 + 1}$.

75. (a) If the x-intercepts are -3, 1 then:

$$\begin{aligned} a = 1: &\quad f(x) = 1(x+3)(x-1) \\ a = 2: &\quad f(x) = 2(x+3)(x-1) \\ a = -2: &\quad f(x) = -2(x+3)(x-1) \\ a = 5: &\quad f(x) = 5(x+3)(x-1) \end{aligned}$$

(b) The value of a does not affect the x-intercepts.

(c) The value of a does not affect the axis of symmetry. Multiplying by a stretches the graph vertically so the axis of symmetry will remain the same.

(d) The x-coordinate of the vertex is not affected by a; it is always $x = -1$. The y-coordinate of the vertex is $y = -4a$.

(e) The x-coordinate of the vertex ($x = -1$) is equal to the midpoint of the x-intercepts (the intercepts are at $x = 1$ and $x = -3$ so the midpoint is $x = -1$).