Math 121 – Section 3.5 Solutions

4. Using the given figure:

   (a) The solution to $g(x) < 0$ is $\boxed{x < -1 \text{ or } x > 4}$.

   (b) The solution to $g(x) \geq 0$ is $\boxed{-1 \leq x \leq 4}$.

5. Using the given figure:

   (a) The solution to $g(x) \geq f(x)$ is $\boxed{-2 \leq x \leq 1}$.

   (b) The solution to $f(x) > g(x)$ is $\boxed{x < -2 \text{ or } x > 1}$.

7. To solve $f(x) = x^2 - 3x - 10 < 0$, we find the x-intercepts of $f(x)$:

$$\begin{aligned} x^2 - 3x - 10 &= 0 \\ (x-5)(x+2) &= 0 \\ x &= 5, \ x = -2 \end{aligned}$$

   Since $f(x)$ opens up and $f(x) < 0$, the solution is $\boxed{-2 < x < 5}$.

12. To solve $f(x) = x^2 - 1 < 0$, we find the x-intercepts of $f(x)$:

$$\begin{aligned} x^2 - 1 &= 0 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

   Since $f(x)$ opens up and $f(x) < 0$, the solution is $\boxed{-1 < x < 1}$.

17. To solve $x(x-7) > 8$, we first rewrite the inequality:

$$\begin{aligned} x(x-7) &> 8 \\ x^2 - 7x &> 8 \\ x^2 - 7x - 8 &> 0 \end{aligned}$$

   The x-intercepts of $f(x) = x^2 - 7x - 8$ are:

$$\begin{aligned} x^2 - 7x - 8 &= 0 \\ (x-8)(x+1) &= 0 \\ x &= 8, \ x = -1 \end{aligned}$$

   Since $f(x)$ opens up and $f(x) > 0$, the solution is $\boxed{x < -1 \text{ or } x > 8}$.

22. To solve $2(2x^2 - 3x) > -9$, we first rewrite the inequality:

$$\begin{aligned} 2(2x^2 - 3x) &> -9 \\ 4x^2 - 6x &> -9 \\ 4x^2 - 6x + 9 &> 0 \end{aligned}$$

The function $f(x) = 4x^2 - 6x + 9$ has no x-intercepts. Since $f(x)$ opens up and $f(x) > 0$, the solution is $\boxed{\text{all real numbers}}$.

25. $f(x) = x^2 - 1$, $g(x) = 3x + 3$

(a) $f(x) = 0$: $\boxed{x = \pm 1}$

(b) $g(x) = 0$: $\boxed{x = -1}$

(c) $f(x) = g(x)$:

$$\begin{aligned} f(x) &= g(x) \\ x^2 - 1 &= 3x + 3 \\ x^2 - 3x - 4 &= 0 \\ (x - 4)(x + 1) &= 0 \\ x = 4,\ x &= -1 \end{aligned}$$

The solutions are $\boxed{x = 4,\ x = -2}$.

(d) $f(x) > 0$: $\boxed{x < -1 \text{ or } x > 1}$

(e) $g(x) \leq 0$: $\boxed{x \leq -1}$

(f) $f(x) > g(x)$: $\boxed{x < -1 \text{ or } x > 4}$

(g) $f(x) \geq 1$:

$$\begin{aligned} f(x) &\geq 1 \\ x^2 - 1 &\geq 1 \\ x^2 - 2 &\geq 0 \\ (x - \sqrt{2})(x + \sqrt{2}) &\geq 0 \end{aligned}$$

The solution is $\boxed{x \leq -\sqrt{2} \text{ or } x \geq \sqrt{2}}$.

30. $f(x) = x^2 - 2x + 1$, $g(x) = -x^2 + 1$

(a) $f(x) = 0$: $\boxed{x = 1}$

(b) $g(x) = 0$: $\boxed{x = \pm 1}$

(c) $f(x) = g(x)$:

$$\begin{aligned} f(x) &= g(x) \\ x^2 - 2x + 1 &= -x^2 + 1 \\ 2x^2 - 2x &= 0 \\ x^2 - x &= 0 \\ x(x - 1) &= 0 \\ x = 0,\ x &= 1 \end{aligned}$$

The solutions are $\boxed{x = 0,\ x = 1}$.

(d) $f(x) > 0$: $\boxed{\text{all } x \text{ except } x = 1}$

(e) $g(x) \leq 0$: $\boxed{-1 \leq x \leq 1}$

(f) $f(x) > g(x)$: $\boxed{x < 0 \text{ or } x > 1}$

(g) $f(x) \geq 1$:

$$\begin{aligned} f(x) &\geq 1 \\ x^2 - 2x + 1 &\geq 1 \\ x^2 - 2x &\geq 0 \\ x(x-2) &\geq 0 \end{aligned}$$

The solution is $\boxed{x \leq 0 \text{ or } x \geq 2}$.

33. $s(t) = 80t - 16t^2$

(a) When the ball strikes the ground, $s(t) = 0$:

$$\begin{aligned} 80t - 16t^2 &= 0 \\ 5t - t^2 &= 0 \\ t(5-t) &= 0 \\ t = 0,\ t &= 5 \end{aligned}$$

The ball strikes the ground after 5 seconds.

(b) When the ball is more than 96 feet above the ground, $s(t) > 96$:

$$\begin{aligned} 80t - 16t^2 &> 96 \\ -16t^2 + 80t - 96 &> 0 \\ t^2 - 5t + 6 &< 0 \\ (t-2)(t-3) &< 0 \end{aligned}$$

Therefore, the ball is more than 96 feet above the ground when $2 < t < 3$.