Math 121 – Section 4.4 Solutions

3. Solve $f(x) = (x-5)^2(x+2) < 0$.

Interval	$(-\infty, -2)$	$(-2, 5)$	$(5, \infty)$
Number Chosen	-3	0	6
Value of f	$f(-3) = -64$	$f(0) = 50$	$f(6) = 8$
Conclusion	negative	positive	positive

The solution is $x < -2$. In interval notation, the solution is $(-\infty, -2)$.

5. Solve $f(x) = x^3 - 4x^2 = x^2(x-4) > 0$.

Interval	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
Number Chosen	-1	1	5
Value of f	$f(-1) = -5$	$f(1) = -3$	$f(5) = 25$
Conclusion	negative	negative	positive

The solution is $x > 4$. In interval notation, the solution is $(4, \infty)$.

11. Solve $f(x) = (x-1)(x^2+x+4) \geq 0$.

First, note that $x^2 + x + 4 = 0$ has no real solutions since the discriminant is negative:

$$b^2 - 4ac = 1^2 - 4(1)(4) = -15$$

Therefore, the only real zero of $f(x)$ is $x = 1$.

Interval	$(-\infty, 1)$	$(1, \infty)$
Number Chosen	0	2
Value of f	$f(0) = -4$	$f(2) = 10$
Conclusion	negative	positive

The solution is $x \geq 1$. In interval notation, the solution is $[1, \infty)$.

18. Solve $x^4 < 9x^2$.

First, rewrite the inequality and factor:

$$x^4 < 9x^2$$
$$x^4 - 9x^2 < 0$$
$$x^2(x^2 - 9) < 0$$
$$f(x) = x^2(x+3)(x-3) < 0$$

Therefore, the real zeros of $f(x)$ are $x = -3, 0, 3$.

Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 3)$	$(3, \infty)$
Number Chosen	-4	-1	1	4
Value of f	$f(-4) = 112$	$f(-1) = -8$	$f(1) = -8$	$f(4) = 112$
Conclusion	positive	negative	negative	positive

The solution is $-3 < x < 0$ or $0 < x < 3$. In interval notation, the solution is $(-3, 0) \cup (0, 3)$.

20. Solve $x^3 > 1$.

First, rewrite the inequality and factor:

$$x^3 > 1$$
$$x^3 - 1 > 0$$
$$f(x) = (x-1)(x^2 + x + 1) > 0$$

Therefore, the real zero of $f(x)$ is $x = 1$ since $x^2 + x + 1 = 0$ has no real solutions (the discriminant is negative).

Interval	$(-\infty, 1)$	$(1, \infty)$
Number Chosen	0	2
Value of f	$f(0) = -1$	$f(2) = 7$
Conclusion	negative	positive

The solution is $x > 1$. In interval notation, the solution is $(1, \infty)$.

21. Solve $f(x) = \dfrac{x+1}{x-1} > 0$.

First, the real zeros of the numerator and denominator of $f(x)$ are $x = -1, 1$.

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Number Chosen	-2	0	2
Value of f	$f(-2) = \frac{1}{3}$	$f(0) = -1$	$f(2) = 3$
Conclusion	positive	negative	positive

The solution is $x < -1$ or $x > 1$. In interval notation, the solution is $(-\infty, -1) \cup (1, \infty)$.

23. Solve $f(x) = \dfrac{(x-1)(x+1)}{x} \leq 0$.
First, the real zeros of the numerator and denominator of $f(x)$ are $x = -1, 0, 1$.

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Number Chosen	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
Value of f	$f(-2) = -\frac{3}{2}$	$f(-\frac{1}{2}) = \frac{3}{2}$	$f(\frac{1}{2}) = -\frac{3}{2}$	$f(2) = \frac{3}{2}$
Conclusion	negative	positive	negative	positive

The solution is $x \leq -1$ or $0 < x \leq 1$. In interval notation, the solution is $(-\infty, -1] \cup (0, 1]$.

27. Solve $6x - 5 < \frac{6}{x}$.
First, rewrite the inequality:

$$\begin{aligned} 6x - 5 &< \frac{6}{x} \\ 6x - 5 - \frac{6}{x} &< 0 \\ \frac{x(6x-5) - 6}{x} &< 0 \\ \frac{6x^2 - 5x - 6}{x} &< 0 \\ f(x) = \frac{(3x+2)(2x-3)}{x} &< 0 \end{aligned}$$

The real zeros of the numerator and denominator of $f(x)$ are $x = -\frac{2}{3}, 0, \frac{3}{2}$.

Interval	$(-\infty, -\frac{2}{3})$	$(-\frac{2}{3}, 0)$	$(0, \frac{3}{2})$	$(\frac{3}{2}, \infty)$
Number Chosen	-1	$-\frac{1}{2}$	1	2
Value of f	$f(-1) = -5$	$f(-\frac{1}{2}) = 4$	$f(1) = -5$	$f(2) = 4$
Conclusion	negative	positive	negative	positive

The solution is $x < -1$ or $-\frac{2}{3} < x < 0$. In interval notation, the solution is $(-\infty, -1) \cup (-\frac{2}{3}, 0)$.