Math 121 – Section 4.5 Solutions

12. The remainder when $f(x) = -4x^3 + 5x^2 + 8$ is divided by $x + 3$ is:

$$f(-3) = -4(-3)^3 + 5(-3)^2 + 8 = \boxed{161}$$

14. The remainder when $f(x) = 4x^4 - 15x^2 - 4$ is divided by $x - 2$ is:

$$f(2) = 4(2)^4 - 15(2)^2 - 4 = \boxed{0}$$

21. $f(x) = -4x^7 + x^3 - x^2 + 2$

- $f(x)$ has at most 7 real zeros since the degree is 7
- there are 3 sign changes in $f(x) \Rightarrow f(x)$ has either 3 or 1 positive real zeros
- since $f(-x) = 4x^7 - x^3 - x^2 + 2$, there are 2 sign changes in $f(-x) \Rightarrow f(x)$ has either 2 or 0 negative real zeros

30. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$

- $f(x)$ has at most 5 real zeros since the degree is 5
- there are 5 sign changes in $f(x) \Rightarrow f(x)$ has either 5, 3, or 1 positive real zeros
- since $f(-x) = -x^5 - x^4 - x^3 - x^2 - 1$, there are 0 sign changes in $f(-x) \Rightarrow f(x)$ has 0 negative real zeros

34. Given $f(x) = x^5 - x^4 + 2x^2 + 3$ we have $a_0 = 3$ and $a_5 = 1$. The factors of a_0 are $p = \pm 1, \pm 3$. The factors of a_5 are $q = \pm 1$. Therefore, the potential rational real zeros of $f(x)$ are:

$$\boxed{\frac{p}{q} = \pm 1, \pm 3}$$

38. Given $f(x) = 6x^4 - x^2 + 2$ we have $a_0 = 2$ and $a_4 = 6$. The factors of a_0 are $p = \pm 1, \pm 2$. The factors of a_4 are $q = \pm 1, \pm 2, \pm 3, \pm 6$. Therefore, the potential rational real zeros of $f(x)$ are:

$$\boxed{\frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3}}$$

45. Given $f(x) = x^3 + 2x^2 - 5x - 6$ we have $a_0 = -6$ and $a_3 = 1$. The factors of a_0 are $p = \pm 1, \pm 2, \pm 3, \pm 6$. The factors of a_3 are $q = \pm 1$. Therefore, the potential rational real zeros of $f(x)$ are:

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$$

Checking some of these we have:

$$\begin{aligned} f(1) &= 1+2-5-6=-8 \\ f(-1) &= -1+2+5-6=0 \\ f(2) &= 8+8-10-6=0 \\ f(-2) &= -8+8+10-6=4 \\ f(3) &= 27+18-15-6=24 \\ f(-3) &= -27+18+15-6=0 \end{aligned}$$

Therefore, since $x=-1,2,-3$ are the real zeros, $f(x)$ is factored as follows:

$$\boxed{f(x)=(x+1)(x-2)(x+3)}$$

52. Given $f(x)=2x^4-x^3-5x^2+2x+2$ we have $a_0=2$ and $a_4=2$. The factors of a_0 are $p=\pm1,\pm2$. The factors of a_4 are $q=\pm1,\pm2$. Therefore, the potential rational real zeros of $f(x)$ are:

$$\frac{p}{q}=\pm1,\pm2,\pm\frac{1}{2}$$

Checking some of these we have:

$$\begin{aligned} f(1) &= 2-1-5+2+2=0 \\ f(-1) &= 2+1-5-2+2=-2 \\ f(2) &= 32-8-20+4+2=10 \\ f(-2) &= 32+8-20-4+2=18 \\ f\left(\frac{1}{2}\right) &= \frac{1}{8}-\frac{1}{8}-\frac{5}{4}+1+2=\frac{7}{4} \\ f\left(-\frac{1}{2}\right) &= \frac{1}{8}+\frac{1}{8}-\frac{5}{4}-1+2=0 \end{aligned}$$

Therefore, since $x=1,-\frac{1}{2}$ are the rational real zeros, $f(x)$ is factored as follows:

$$f(x)=(x-1)(2x+1)q(x)=(2x^2-x-1)q(x)$$

To find $q(x)$ we use long division:

$$\begin{array}{r} x^2 \qquad -2 \\ 2x^2-x-1 \,\overline{)\; 2x^4-x^3-5x^2+2x+2} \\ \underline{-2x^4+x^3\;\;+x^2\qquad\quad} \\ -4x^2+2x+2 \\ \underline{4x^2-2x-2} \\ 0 \end{array}$$

Solving $q(x)=x^2-2=0$ we have:

$$\begin{aligned} q(x)=x^2-2&=0 \\ x^2&=2 \\ x&=\pm\sqrt{2} \end{aligned}$$

Therefore, $f(x)$ is factored as follows:

$$\boxed{f(x)=(x-1)(2x+1)(x-\sqrt{2})(x+\sqrt{2})}$$

57. To solve $f(x) = x^4 - x^3 + 2x^2 - 4x - 8 = 0$ we look for the rational real zeros. The factors of $a_0 = -8$ are $p = \pm1, \pm2, \pm4, \pm8$. The factors of $a_4 = 1$ are $q = \pm1$. Therefore, the potential rational real zeros of $f(x)$ are:

$$\frac{p}{q} = \pm1, \pm2, \pm4, \pm8$$

Checking some of these we have:

$$\begin{aligned} f(1) &= 1 - 1 + 2 - 4 - 8 = -12 \\ f(-1) &= 1 + 1 + 2 + 4 - 8 = 0 \\ f(2) &= 16 - 8 + 8 - 8 - 8 = 0 \end{aligned}$$

Therefore, since $x = -1, 2$ are rational real zeros, $f(x)$ is factored as follows:

$$f(x) = (x + 1)(x - 2)q(x) = (x^2 - x - 2)q(x)$$

To find $q(x)$ we use long division:

$$\begin{array}{r|l} & x^2 \qquad\quad + 4 \\ \hline x^2 - x - 2 & x^4 - x^3 + 2x^2 - 4x - 8 \\ & -x^4 + x^3 + 2x^2 \\ \hline & \qquad\qquad 4x^2 - 4x - 8 \\ & \qquad\quad -4x^2 + 4x + 8 \\ \hline & \qquad\qquad\qquad\qquad 0 \end{array}$$

Therefore, $f(x)$ is factored as follows:

$$\boxed{f(x) = (x + 1)(x - 2)(x^2 + 4)}$$

62. To solve $f(x) = 2x^3 - 11x^2 + 10x + 8 = 0$ we look for the rational real zeros. The factors of $a_0 = 8$ are $p = \pm1, \pm2, \pm4, \pm8$. The factors of $a_3 = 2$ are $q = \pm1, \pm2$. Therefore, the potential rational real zeros of $f(x)$ are:

$$\frac{p}{q} = \pm1, \pm2, \pm4, \pm8, \pm\frac{1}{2}$$

Checking some of these we have:

$$\begin{aligned} f(1) &= 2 - 11 + 10 + 8 = 9 \\ f(-1) &= -2 - 11 - 10 + 8 = -15 \\ f(2) &= 16 - 44 + 20 + 8 = 0 \\ f(-2) &= -16 - 44 - 20 + 8 = -72 \\ f(4) &= 128 - 176 + 40 + 8 = 0 \\ f(-4) &= -128 - 176 - 40 + 8 = -336 \\ f\left(\frac{1}{2}\right) &= \frac{1}{4} - \frac{11}{4} + 5 + 8 = \frac{21}{2} \\ f\left(-\frac{1}{2}\right) &= -\frac{1}{4} - \frac{11}{4} - 5 + 8 = 0 \end{aligned}$$

Therefore, since $x = 2, 4, -\frac{1}{2}$ are the real zeros, $f(x)$ is factored as follows:

$$\boxed{f(x) = (x - 2)(x - 4)(2x + 1)}$$