Math 121 – Section 5.2 Solutions

13. The function is not one-to-one since the inputs $x = 2, -3$ have the same output $y = 6$.

19. The function is not one-to-one. The graph does not pass the horizontal line test.

22. The function is not one-to-one. The graph does not pass the horizontal line test (there are infinitely many intersections of the graph with the line $y = 2$).

31. $f(x) = 3x + 4$, $g(x) = \dfrac{1}{3}(x - 4)$

$$\begin{aligned} f(g(x)) &= f\left(\frac{1}{3}(x-4)\right) \\ &= 3 \cdot \frac{1}{3}(x-4) + 4 \\ &= x - 4 + 4 \\ &= x \\ g(f(x)) &= g(3x+4) \\ &= \frac{1}{3}(3x + 4 - 4) \\ &= x \end{aligned}$$

40. $f(x) = \dfrac{x-5}{2x+3}$, $g(x) = \dfrac{3x+5}{1-2x}$

$$\begin{aligned} f(g(x)) &= f\left(\frac{3x+5}{1-2x}\right) \\ &= \frac{\frac{3x+5}{1-2x} - 5}{2 \cdot \frac{3x+5}{1-2x} + 3} \\ &= \frac{3x + 5 - 5(1-2x)}{2(3x+5) + 3(1-2x)} \\ &= \frac{13x}{13} \\ &= x \\ g(f(x)) &= g\left(\frac{x-5}{2x+3}\right) \\ &= \frac{3 \cdot \frac{x-5}{2x+3} + 5}{1 - 2 \cdot \frac{x-5}{2x+3}} \\ &= \frac{3(x-5) + 5(2x+3)}{2x + 3 - 2(x-5)} \\ &= \frac{13x}{13} \\ &= x \end{aligned}$$

41.

47. The inverse of $f(x) = 3x$ is found as follows:

- Let $y = 3x$.
- Switch x and y: $x = 3y$.
- Solve for y in terms of x: $y = \dfrac{x}{3}$.

The inverse is $f^{-1}(x) = \dfrac{x}{3}$. The domain and range of both f and f^{-1} are all real numbers.

58. The inverse of $f(x) = \dfrac{4}{x+2}$ is found as follows:

- Let $y = \dfrac{4}{x+2}$.
- Switch x and y: $x = \dfrac{4}{y+2}$.
- Solve for y in terms of x: $y = \dfrac{4}{x} - 2$.

The inverse is $f^{-1}(x) = \dfrac{4}{x} - 2$. The domain of f (and the range of f^{-1}) is all real numbers except -2. The range of f (and the domain of f^{-1}) is all real numbers except 0.

60. The inverse of $f(x) = \dfrac{4}{2-x}$ is found as follows:

- Let $y = \dfrac{4}{2-x}$.
- Switch x and y: $x = \dfrac{4}{2-y}$.
- Solve for y in terms of x: $y = 2 - \dfrac{4}{x}$.

The inverse is $f^{-1}(x) = 2 - \dfrac{4}{x}$. The domain of f (and the range of f^{-1}) is all real numbers except 2. The range of f (and the domain of f^{-1}) is all real numbers except 0.

65. The inverse of $f(x) = \dfrac{3x+4}{2x-3}$ is found as follows:

- Let $y = \dfrac{3x+4}{2x-3}$.
- Switch x and y: $x = \dfrac{3y+4}{2y-3}$.
- Solve for y in terms of x:

$$\begin{aligned} x &= \frac{3y+4}{2y-3} \\ x(2y-3) &= 3y+4 \\ 2xy-3x &= 3y+4 \\ 2xy-3y &= 3x+4 \\ y(2x-3) &= 3x+4 \\ y &= \frac{3x+4}{2x-3} \end{aligned}$$

The inverse is $f^{-1}(x) = \dfrac{3x+4}{2x-3}$. The domain of f (and the range of f^{-1}) is all real numbers except $\frac{3}{2}$. The range of f (and the domain of f^{-1}) is all real numbers except $\frac{3}{2}$.