Math 121 – Section 7.4 Solutions

9. $\sin\frac{5\pi}{12}$

$$\begin{aligned}\sin\frac{5\pi}{12} &= \sin\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)\\ &= \sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)\\ &= \sin\frac{\pi}{6}\cos\frac{\pi}{4}+\cos\frac{\pi}{6}\sin\frac{\pi}{4}\\ &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\\ &= \frac{\sqrt{2}+\sqrt{6}}{4}\end{aligned}$$

17. $\sin\frac{17\pi}{12}$

$$\begin{aligned}\sin\frac{17\pi}{12} &= \sin\left(\frac{8\pi}{12}+\frac{9\pi}{12}\right)\\ &= \sin\left(\frac{2\pi}{3}+\frac{3\pi}{4}\right)\\ &= \sin\frac{2\pi}{3}\cos\frac{3\pi}{4}+\cos\frac{2\pi}{3}\sin\frac{3\pi}{4}\\ &= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)+\left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)\\ &= \frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$

31. $\sin\alpha=\frac{3}{5},\ 0<\alpha<\frac{\pi}{2};\quad \cos\beta=\frac{2\sqrt{5}}{5},\ -\frac{\pi}{2}<\beta<0$

Using the Pythagorean Identity or a right triangle, we find that:

$$\cos\alpha=\frac{4}{5},\quad \sin\beta=-\frac{\sqrt{5}}{5}$$

Then

(a) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta=\left(\frac{3}{5}\right)\left(\frac{2\sqrt{5}}{5}\right)+\left(\frac{4}{5}\right)\left(-\frac{\sqrt{5}}{5}\right)=\frac{2\sqrt{5}}{25}$

(b) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\frac{11\sqrt{5}}{25}$

(c) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta=\frac{10\sqrt{5}}{25}$

(d) $\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=2$

37. $\sin\theta = \frac{1}{3}$, θ is in Quadrant II

(a) $\cos\theta = -\frac{2\sqrt{2}}{3}$

(b) $\sin\left(\theta + \frac{\pi}{6}\right) = \sin\theta\cos\frac{\pi}{6} + \cos\theta\sin\frac{\pi}{6} = \left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{2\sqrt{2}}{3}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}-2\sqrt{2}}{6}$

(c) $\cos\left(\theta - \frac{\pi}{3}\right) = \frac{\sqrt{3}-2\sqrt{2}}{6}$

(d) $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{4-\sqrt{2}}{4+\sqrt{2}} = \frac{7-4\sqrt{2}}{9}$

45. Establish $\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$.

$$\begin{aligned}\sin\left(\frac{\pi}{2} + \theta\right) &= \sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta \\ &= (1)\cos\theta + (0)\sin\theta \\ &= \cos\theta\end{aligned}$$

49. Establish $\sin(\pi + \theta) = -\sin\theta$.

$$\begin{aligned}\sin(\pi + \theta) &= \sin\pi\cos\theta + \cos\pi\sin\theta \\ &= (0)\cos\theta + (-1)\sin\theta \\ &= -\sin\theta\end{aligned}$$

55. Establish $\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$.

$$\begin{aligned}\sin(\alpha+\beta) + \sin(\alpha-\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ &= 2\sin\alpha\cos\beta\end{aligned}$$

73. Find the exact value of $\sin\left[\sin^{-1}\frac{3}{5} - \cos^{-1}\left(-\frac{4}{5}\right)\right]$.

Let $\alpha = \sin^{-1}\frac{3}{5}$ and $\beta = \cos^{-1}\left(-\frac{4}{5}\right)$. Then,

$$\sin\alpha = \frac{3}{5}, \quad 0 < \alpha < \frac{\pi}{2}; \quad \cos\beta = -\frac{4}{5}, \quad \frac{\pi}{2} < \beta < \pi$$

Using a Pythagorean Identity or a right triangle, we find that:

$$\cos\alpha = \frac{4}{5}, \qquad \sin\beta = \frac{3}{5}$$

Therefore,

$$\begin{aligned}\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ &= \left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) - \left(\frac{4}{5}\right)\left(\frac{3}{5}\right) \\ &= -\frac{24}{25}\end{aligned}$$