Math 121 – Section 7.5 Solutions

7. $\sin\theta = \dfrac{3}{5}, \quad 0 < \theta < \dfrac{\pi}{2}$

Using a right triangle or an identity we find that $\cos\theta = \dfrac{4}{5}$.

(a) $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\dfrac{3}{5}\right)\left(\dfrac{4}{5}\right) = \dfrac{24}{25}$

(b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\dfrac{4}{5}\right)^2 - \left(\dfrac{3}{5}\right)^2 = \dfrac{7}{25}$

(c) $\sin\dfrac{\theta}{2} = \sqrt{\dfrac{1-\cos\theta}{2}} = \sqrt{\dfrac{1-\frac{4}{5}}{2}} = \dfrac{1}{\sqrt{10}}$ where we use the positive root since $0 < \dfrac{\theta}{2} < \dfrac{\pi}{4}$

(d) $\cos\dfrac{\theta}{2} = \sqrt{\dfrac{1+\cos\theta}{2}} = \sqrt{\dfrac{1+\frac{4}{5}}{2}} = \dfrac{3}{\sqrt{10}}$

10. $\tan\theta = \dfrac{1}{2}, \quad \pi < \theta < \dfrac{3\pi}{2}$

Using a right triangle or an identity we find that $\sin\theta = -\dfrac{1}{\sqrt{5}}, \quad \cos\theta = -\dfrac{2}{\sqrt{5}}$

(a) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{4}{5}$

(b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \dfrac{3}{5}$

(c) $\sin\dfrac{\theta}{2} = \sqrt{\dfrac{1-\cos\theta}{2}} = \sqrt{\dfrac{5+\sqrt{5}}{10}}$ where we use the positive root since $\dfrac{\pi}{2} < \dfrac{\theta}{2} < \dfrac{3\pi}{4}$

(d) $\cos\dfrac{\theta}{2} = -\sqrt{\dfrac{1+\cos\theta}{2}} = -\sqrt{\dfrac{5-\sqrt{5}}{10}}$

19. We compute the exact value of $\sin 22.5^\circ$ as follows:

$$\begin{aligned}
\sin 22.5^\circ &= \sin\frac{45^\circ}{2} \\
&= \sqrt{\frac{1-\cos 45^\circ}{2}} \\
&= \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} \\
&= \frac{\sqrt{\sqrt{2}-1}}{2}
\end{aligned}$$

20. We compute the exact value of $\cos 22.5^\circ$ as follows:

$$\begin{aligned}\cos 22.5^\circ &= \cos\frac{45^\circ}{2}\\ &= \sqrt{\frac{1+\cos 45^\circ}{2}}\\ &= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}\\ &= \frac{\sqrt{\sqrt{2}+1}}{2}\end{aligned}$$

48. Establish the identity $\dfrac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos(2\theta)$

$$\begin{aligned}\frac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} &= \frac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}}\\ &= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}\\ &= \frac{\cos 2\theta}{1}\\ &= \cos 2\theta\end{aligned}$$

69. Find the exact value of $\sin\left(2\sin^{-1}\dfrac{1}{2}\right)$.

$$\sin\left(2\sin^{-1}\frac{1}{2}\right) = \sin\left(2\cdot\frac{\pi}{6}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

71. Find the exact value of $\cos\left(2\sin^{-1}\dfrac{3}{5}\right)$.

Let $\theta = \sin^{-1}\dfrac{3}{5}$. Then $\sin\theta = \dfrac{3}{5}$ and $\cos\theta = \dfrac{4}{5}$ and:

$$\cos\left(2\sin^{-1}\frac{3}{5}\right) = \cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{7}{25}$$