Math 121 – Section 7.8 Solutions

5. Solve $2\cos^2\theta + \cos\theta = 0$ on the interval $0 \le \theta < 2\pi$.

$$\begin{aligned} 2\cos^2\theta + \cos\theta &= 0 \\ \cos\theta(2\cos\theta + 1) &= 0 \\ \Rightarrow \quad \cos\theta = 0,\ \cos\theta &= -\frac{1}{2} \end{aligned}$$

The solutions are $\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}}$ and $\boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}}$.

7. Solve $2\sin^2\theta - \sin\theta - 1 = 0$ on the interval $0 \le \theta < 2\pi$.

$$\begin{aligned} 2\sin^2\theta - \sin\theta - 1 &= 0 \\ (2\sin\theta + 1)(\sin\theta - 1) &= 0 \\ \Rightarrow \quad \sin\theta = -\frac{1}{2},\ \sin\theta &= 1 \end{aligned}$$

The solutions are $\boxed{\theta = \frac{7\pi}{6}, \frac{11\pi}{6}}$ and $\boxed{\theta = \frac{\pi}{2}}$.

13. Solve $\sin^2\theta = 6(\cos\theta + 1)$ on the interval $0 \le \theta < 2\pi$.

$$\begin{aligned} \sin^2\theta &= 6(\cos\theta + 1) \\ 1 - \cos^2\theta &= 6\cos\theta + 6 \\ \cos^2\theta + 6\cos\theta + 5 &= 0 \\ (\cos\theta + 1)(\cos\theta + 5) &= 0 \\ \Rightarrow \quad \cos\theta = -1,\ \cos\theta &= -5 \end{aligned}$$

The solution is $\boxed{\theta = \pi}$. Note that there are no solutions to the second equation since $-1 \le \cos\theta \le 1$.

17. Solve $\cos\theta = \sin\theta$ on the interval $0 \le \theta < 2\pi$.

The solutions are $\boxed{\theta = \frac{\pi}{4}, \frac{5\pi}{4}}$.

23. Solve $\cos 2\theta = \cos\theta$ on the interval $0 \le \theta < 2\pi$.

$$\begin{aligned} \cos 2\theta &= \cos\theta \\ 2\cos^2\theta - 1 &= \cos\theta \\ 2\cos^2\theta - \cos\theta - 1 &= 0 \\ (2\cos\theta + 1)(\cos\theta - 1) &= 0 \\ \Rightarrow \quad \cos\theta = -\frac{1}{2},\ \cos\theta &= 1 \end{aligned}$$

The solutions are $\boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}}$ and $\boxed{\theta = 0}$.

31. Solve $2\sin^2\theta - 5\sin\theta + 3 = 0$ on the interval $0 \leq \theta < 2\pi$.

$$\begin{aligned} 2\sin^2\theta - 5\sin\theta + 3 &= 0 \\ (2\sin\theta - 3)(\sin\theta - 1) &= 0 \\ \Rightarrow \quad \sin\theta = \frac{3}{2},\ \sin\theta &= 1 \end{aligned}$$

The solution is $\boxed{\theta = \frac{\pi}{2}}$. Note that there are no solutions to the first equation since $-1 \leq \sin\theta \leq 1$.