Math 121 – Section 8.2 Solutions

9. $A = 40°$. Using the Law of Sines, we have:

$$\begin{aligned} \frac{a}{\sin A} &= \frac{c}{\sin C} \\ \frac{a}{\sin 40°} &= \frac{5}{\sin 95°} \\ a &= \frac{5 \sin 40°}{\sin 95°} \\ a &\approx 3.23 \end{aligned}$$

and

$$\begin{aligned} \frac{b}{\sin B} &= \frac{c}{\sin C} \\ \frac{b}{\sin 45°} &= \frac{5}{\sin 95°} \\ b &= \frac{5 \sin 45°}{\sin 95°} \\ b &\approx 3.55 \end{aligned}$$

11. $B = 45°$. Using the Law of Sines, we have:

$$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} \\ \frac{a}{\sin 50°} &= \frac{3}{\sin 45°} \\ a &= \frac{3 \sin 50°}{\sin 45°} \\ a &\approx 3.25 \end{aligned}$$

and

$$\begin{aligned} \frac{c}{\sin C} &= \frac{b}{\sin B} \\ \frac{c}{\sin 85°} &= \frac{3}{\sin 45°} \\ b &= \frac{3 \sin 85°}{\sin 45°} \\ b &\approx 4.23 \end{aligned}$$

17. $C = 120°$. Using the Law of Sines, we have:

$$\begin{aligned} \frac{b}{\sin B} &= \frac{a}{\sin A} \\ \frac{b}{\sin 20°} &= \frac{2}{\sin 40°} \\ a &= \frac{2 \sin 20°}{\sin 40°} \\ a &\approx 1.06 \end{aligned}$$

and

$$\begin{aligned}
\frac{c}{\sin C} &= \frac{a}{\sin A} \\
\frac{c}{\sin 120^\circ} &= \frac{2}{\sin 40^\circ} \\
b &= \frac{2\sin 120^\circ}{\sin 40^\circ} \\
b &\approx 2.69
\end{aligned}$$

21. $B = 40^\circ$. Using the Law of Sines, we have:

$$\begin{aligned}
\frac{a}{\sin A} &= \frac{c}{\sin C} \\
\frac{a}{\sin 110^\circ} &= \frac{3}{\sin 30^\circ} \\
a &= \frac{3\sin 110^\circ}{\sin 30^\circ} \\
a &\approx 5.64
\end{aligned}$$

and

$$\begin{aligned}
\frac{b}{\sin B} &= \frac{c}{\sin C} \\
\frac{b}{\sin 40^\circ} &= \frac{3}{\sin 30^\circ} \\
b &= \frac{3\sin 40^\circ}{\sin 30^\circ} \\
b &\approx 3.86
\end{aligned}$$

25. Using the Law of Sines, we have:

$$\begin{aligned}
\frac{\sin B}{b} &= \frac{\sin A}{a} \\
\frac{\sin B}{2} &= \frac{\sin 50^\circ}{3} \\
\sin B &= \frac{2\sin 50^\circ}{3} \\
\sin B &\approx 0.511
\end{aligned}$$

Two (approximate) solutions to the above equation are $B = 30.71^\circ, 149.29^\circ$. In the first case, we find that $C \approx 99.29^\circ$. In the second case, we find that $C \approx -19.29^\circ$ which is impossible. Therefore, there is only one triangle.

The remaining side c is found using the Law of Sines:

$$\begin{aligned}
\frac{c}{\sin C} &= \frac{a}{\sin A} \\
\frac{c}{\sin 99.29^\circ} &= \frac{3}{\sin 50^\circ} \\
c &= \frac{3\sin 99.29^\circ}{\sin 50^\circ} \\
c &\approx 3.86
\end{aligned}$$

31. Using the Law of Sines, we have:

$$\begin{aligned}
\frac{\sin C}{c} &= \frac{\sin B}{b} \\
\frac{\sin C}{6} &= \frac{\sin 20^\circ}{4} \\
\sin C &= \frac{6 \sin 20^\circ}{4} \\
\sin C &\approx 0.513
\end{aligned}$$

Two (approximate) solutions to the above equation are $C = 30.87^\circ, 149.13^\circ$. In the first case, we find that $A \approx 129.13^\circ$. In the second case, we find that $A \approx 10.87^\circ$. Therefore, there are two triangles.

The remaining side a is found using the Law of Sines. In the first case, we have:

$$\begin{aligned}
\frac{a}{\sin A} &= \frac{b}{\sin B} \\
\frac{a}{\sin 129.13^\circ} &= \frac{4}{\sin 20^\circ} \\
a &= \frac{4 \sin 129.13^\circ}{\sin 20^\circ} \\
a &\approx 9.07
\end{aligned}$$

In the second case, we have:

$$\begin{aligned}
\frac{a}{\sin A} &= \frac{b}{\sin B} \\
\frac{a}{\sin 10.87^\circ} &= \frac{4}{\sin 20^\circ} \\
a &= \frac{4 \sin 10.87^\circ}{\sin 20^\circ} \\
a &\approx 2.21
\end{aligned}$$

The two sets of solutions are:

$$\begin{aligned}
C = 30.87^\circ,\ A = 129.13^\circ,\ a = 9.087 \\
C = 149.13^\circ,\ A = 10.87^\circ,\ a = 2.21
\end{aligned}$$

39. The angle at P is 155° and the angle at Q is 10°. Using the Law of Sines we have:

$$\begin{aligned}
\frac{d}{\sin 15^\circ} &= \frac{1000}{\sin 10^\circ} \\
d &= \frac{1000 \sin 15^\circ}{\sin 10^\circ} \\
d &\approx 1490 \text{ feet}
\end{aligned}$$

47. Using the Law of Sines, the angle at R is:

$$\begin{aligned}
\frac{\sin R}{123} &= \frac{\sin 60^\circ}{184.5} \\
\sin R &= \frac{123 \sin 60^\circ}{184.5} \\
\sin R &\approx 0.577 \\
R &\approx 35.26^\circ
\end{aligned}$$

The angle at P is then 84.74°. The perpendicular distance from R to PQ is:

$$d = 184.5 \sin 84.74^\circ \approx 183.7 \text{ feet}$$