Math 121 – Section 8.4 Solutions

5. The height of the triangle is:

$$h = 2\sin 45^\circ = \sqrt{2}$$

Since the base is $c = 4$, the area is:

$$A = \frac{1}{2}ch = \frac{1}{2}(4)(\sqrt{2}) = \boxed{2\sqrt{2}}$$

7. Using the Law of Cosines, the side c is:

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ c^2 &= 2^2 + 3^2 - 2(2)(3)\cos 95^\circ \\ c^2 &= 4 + 9 - 12\cos 95^\circ \\ c^2 &= 14.045 \\ c &= 3.75 \end{aligned}$$

Using the Law of Sines, the angle B is:

$$\begin{aligned} \frac{\sin B}{b} &= \frac{\sin C}{c} \\ \frac{\sin B}{3} &= \frac{\sin 95^\circ}{3.75} \\ \sin B &= \frac{3}{3.75}\sin 95^\circ \\ \sin B &= 0.797 \\ B &= 52.89^\circ \end{aligned}$$

The height of the triangle is:

$$h = 2\sin 52.89^\circ = 1.59$$

Since the base is $c = 3.75$, the area is:

$$A = \frac{1}{2}ch = \frac{1}{2}(3.75)(1.59) = \boxed{2.99}$$

11. Use Heron's formula with $a = 9$, $b = 6$, and $c = 4$:

$$s = \frac{1}{2}(a + b + c) = \frac{19}{2}$$

Then

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ A &= \sqrt{\frac{19}{2}\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)\left(\frac{11}{2}\right)} \\ A &= \boxed{\frac{\sqrt{1463}}{4}} \end{aligned}$$

15. The height is:

$$h = 1 \sin 80^\circ = 0.98$$

Since the base is $c = 3$, the area is:

$$A = \frac{1}{2}ch = \frac{1}{2}(3)(0.98) = \boxed{1.48}$$

19. By observation, the triangle with sides $a = 12$, $b = 13$, and $c = 5$ is a right triangle. Therefore, the area is:

$$A = \frac{1}{2}ac = \frac{1}{2}(12)(5) = \boxed{30}$$

33. The area of the sector is:

$$A_{\text{sector}} = \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2\left(\frac{70\pi}{180}\right) = \frac{112\pi}{9}$$

The height of the triangle is:

$$h = 8 \sin 70^\circ = 7.52$$

Since the base is 8, the area of the triangle is:

$$A_{\text{triangle}} = \frac{1}{2}(8)(7.52) = 30.07$$

The area of the shaded region is:

$$A = A_{\text{sector}} - A_{\text{triangle}} = \frac{112\pi}{9} - 30.07 = 9.03$$