Math 121 – Section 9.3 Solutions

11. $1+i \Rightarrow x=1,\ y=1$

$$\begin{aligned} r &= \sqrt{x^2+y^2} = \sqrt{1^2+1^2} = \sqrt{2} \\ \cos\theta &= \frac{x}{r} = \frac{1}{\sqrt{2}} \\ \sin\theta &= \frac{y}{r} = \frac{1}{\sqrt{2}} \end{aligned}$$

Using the last two equations, we find that $\theta = \frac{\pi}{4}$. Therefore,

$$\boxed{1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}$$

13. $\sqrt{3}-i \Rightarrow x=\sqrt{3},\ y=-1$

$$\begin{aligned} r &= \sqrt{x^2+y^2} = \sqrt{(\sqrt{3})^2+(-1)^2} = \sqrt{4} = 2 \\ \cos\theta &= \frac{x}{r} = \frac{\sqrt{3}}{2} \\ \sin\theta &= \frac{y}{r} = -\frac{1}{2} \end{aligned}$$

Using the last two equations, we find that $\theta = \frac{11\pi}{6}$. Therefore,

$$\boxed{\sqrt{3}-i = 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)}$$

23. $2(\cos 120^\circ + i\sin 120^\circ) = 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \boxed{-1+i\sqrt{3}}$

25. $4\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = 4\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = \boxed{2\sqrt{2}-2i\sqrt{2}}$

33. $z = 2(\cos 40^\circ + i\sin 40^\circ),\ w = 4(\cos 20^\circ + i\sin 20^\circ)$

$$\begin{aligned} zw &= (2)(4)[\cos(40^\circ+20^\circ) + i\sin(40^\circ+20^\circ)] = 8(\cos 60^\circ + i\sin 60^\circ) \\ \frac{z}{w} &= \frac{2}{4}[\cos(40^\circ-20^\circ) + i\sin(40^\circ-20^\circ)] = \frac{1}{2}(\cos 20^\circ + i\sin 20^\circ) \end{aligned}$$

35. $z = 3(\cos 130° + i\sin 130°)$, $w = 4(\cos 270° + i\sin 270°)$

$zw = (3)(4)[\cos(130° + 270°) + i\sin(130° + 270°)] = 12(\cos 400° + i\sin 400°) = 12(\cos 40° + i\sin 40°)$

$\dfrac{z}{w} = \dfrac{3}{4}[\cos(130° - 270°) + i\sin(130° - 270°)] = \dfrac{3}{4}(\cos(-140°) + i\sin(-140°)) = \dfrac{3}{4}(\cos 220° + i\sin 220°)$

Note that we converted 400° into 40° by subtracting 360° and we converted −140° into 220° by adding 360°. This was necessary because polar form requires that the angle is between 0 and 2π (or 360°).

41. Using DeMoivre's Theorem:

$$\begin{aligned}[4(\cos 40° + i\sin 40°)]^3 &= 4^3[\cos(3\cdot 40°) + i\sin(3\cdot 40°)] \\ &= 64(\cos 120° + i\sin 120°) \\ &= 64\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \\ &= \boxed{-32 + 32i\sqrt{3}}\end{aligned}$$

43. Using DeMoivre's Theorem:

$$\begin{aligned}\left[[2\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)\right]^5 &= 2^5\left[\cos\left(5\cdot\frac{\pi}{10}\right) + i\sin\left(5\cdot\frac{\pi}{10}\right)\right] \\ &= 32\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \\ &= \boxed{32i}\end{aligned}$$

53. $1 + i \Rightarrow r = \sqrt{2}$, $\theta_0 = \dfrac{\pi}{4}$

$$\begin{aligned} z_0 &= \sqrt[3]{\sqrt{2}}\left[\cos\left(\frac{\frac{\pi}{4} + 2\cdot 0\cdot\pi}{3}\right) + i\sin\left(\frac{\frac{\pi}{4} + 2\cdot 0\cdot\pi}{3}\right)\right] = 2^{1/6}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) \\ z_1 &= \sqrt[3]{\sqrt{2}}\left[\cos\left(\frac{\frac{\pi}{4} + 2\cdot 1\cdot\pi}{3}\right) + i\sin\left(\frac{\frac{\pi}{4} + 2\cdot 1\cdot\pi}{3}\right)\right] = 2^{1/6}\left(\cos\frac{9\pi}{12} + i\sin\frac{9\pi}{12}\right) \\ z_2 &= \sqrt[3]{\sqrt{2}}\left[\cos\left(\frac{\frac{\pi}{4} + 2\cdot 2\cdot\pi}{3}\right) + i\sin\left(\frac{\frac{\pi}{4} + 2\cdot 2\cdot\pi}{3}\right)\right] = 2^{1/6}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)\end{aligned}$$

55. $4 - 4\sqrt{3}i \Rightarrow r = 8$, $\theta_0 = \dfrac{5\pi}{3}$

$$\begin{aligned} z_0 &= \sqrt[4]{8}\left[\cos\left(\frac{\frac{5\pi}{3} + 2\cdot 0\cdot\pi}{4}\right) + i\sin\left(\frac{\frac{5\pi}{3} + 2\cdot 0\cdot\pi}{4}\right)\right] = 8^{1/4}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) \\ z_1 &= \sqrt[4]{8}\left[\cos\left(\frac{\frac{5\pi}{3} + 2\cdot 1\cdot\pi}{4}\right) + i\sin\left(\frac{\frac{5\pi}{3} + 2\cdot 1\cdot\pi}{4}\right)\right] = 8^{1/4}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right) \\ z_2 &= \sqrt[4]{8}\left[\cos\left(\frac{\frac{5\pi}{3} + 2\cdot 2\cdot\pi}{4}\right) + i\sin\left(\frac{\frac{5\pi}{3} + 2\cdot 2\cdot\pi}{4}\right)\right] = 8^{1/4}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right) \\ z_3 &= \sqrt[4]{8}\left[\cos\left(\frac{\frac{5\pi}{3} + 2\cdot 3\cdot\pi}{4}\right) + i\sin\left(\frac{\frac{5\pi}{3} + 2\cdot 3\cdot\pi}{4}\right)\right] = 8^{1/4}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)\end{aligned}$$