MATH 181
Final Exam Practice Problems
Summer 2009

1. Evaluate the following integrals:

$$\text{(a)} \int \frac{dx}{x^2 - 5x + 6}, \qquad \text{(b)} \int \frac{x-4}{x^2(x+2)}\, dx$$

2. Determine if the following improper integrals converge or not.

$$\text{(a)} \int_0^\infty xe^{-2x}\, dx, \qquad \text{(b)} \int_0^\infty \frac{dx}{x^2+4}, \qquad \text{(c)} \int_3^4 \frac{dx}{(x-3)^{\frac{99}{100}}}\, dx$$

3. Find the arclength of the graph of the function $y = 2x^{3/2} + 5$ between $x = 0$ and $x = 1$.

4. Compute the arclength of the curve $y = \frac{1}{6}x^3 + \frac{1}{2x}, \quad 2 \le x \le 3.$

5. Find the 5th degree Taylor polynomial of the function $f(x) = \sin(2x)$ centered at $x = 0$.

6. Given that the 3rd degree Taylor Polynomial for a function $f(x)$ centered at $x = 0$ is:

$$T_3(x) = 3 - 2x + 4x^2 - 5x^3$$

   (a) What is $f(0)$?
   (b) What is $f'(0)$?
   (c) What is $f''(0)$?

7. Determine whether the following sequences converge or diverge:

   (a) $\{a_n\} = \left\{\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \ldots\right\}$
   (b) $a_n = \frac{\ln n}{n}$
   (c) $a_n = \frac{n}{\sqrt{n^3+1}}$

8. Find the sums of the following series:

$$\text{(a)} \sum_{n=0}^\infty \frac{2^n - 1}{5^n}, \qquad \text{(b)} \sum_{n=3}^\infty \frac{2 \cdot 3^{n-1}}{5^{n+2}}, \qquad \text{(c)} \sum_{n=1}^\infty \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$$

9. Determine whether the following series converge or not:

$$\text{(a)} \sum_{n=1}^\infty \frac{n+2}{\sqrt{n^3+n+5}}, \qquad \text{(b)} \sum_{n=1}^\infty \frac{(-1)^{n+1}(n^3+1)}{2^n}, \qquad \text{(c)} \sum_{n=1}^\infty \frac{n^2 3^n}{n!}, \qquad \text{(d)} \sum_{n=2}^\infty \frac{1}{n \ln n}$$

10. Determine whether the following series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$$

11. Compute the interval of convergence of the following power series:

$$\text{(a)} \sum_{n=0}^{\infty}\frac{(-2)^n(x+4)^n}{n+3}, \qquad \text{(b)} \sum_{n=0}^{\infty}\frac{3^n(x-1)^{2n}}{n^2}$$

12. Find the radius and interval of convergence for the power series:

$$\sum_{n=1}^{\infty}\frac{2^n x^n}{n^2 3^n}$$

13. Consider the function $f(x) = \sin(2x)$.
    (a) Compute the first three non-zero terms of the Maclaurin Series of $f(x)$.
    (b) Compute the first three non-zero terms of the Taylor Series of $f(x)$ about $x = \pi$.

14. Use the Maclaurin Series for $f(x) = \dfrac{1}{1-x}$:

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$

to find the Maclaurin Series for the function $f(x) = \dfrac{1}{1-2x}$. Write your answer in summation form.

15. The Taylor series for $\sin x$ centered at 0 (the Maclaurin Series) is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

    (a) Write the series for $\sin(t^2)$ (with four nonzero terms).
    (b) Write the series for $\int_0^x \sin(t^2)\,dt$ (with four nonzero terms).
    (c) Write the series for $\int_0^{1/2} \sin(t^2)\,dt$ (with four nonzero terms).