MATH 180
Midterm Practice Problems
Summer 2009

1. Evaluate the following integrals:

   (a) $\displaystyle\int xe^{x^2}\,dx$ (b) $\displaystyle\int \frac{1}{16x^2+1}\,dx$ (c) $\displaystyle\int \frac{t^3}{\sqrt{t^4+9}}\,dt$ (d) $\displaystyle\int \tan^2\theta\sec^2\theta\,d\theta$

   (e) $\displaystyle\int_0^1 x\sqrt{1-x^2}\,dx$ (f) $\displaystyle\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}}\,dx$ (g) $\displaystyle\int_0^1 \frac{4}{\sqrt{1-x^2}}\,dx$

2. Find $\dfrac{dy}{dx}$ for each of the following:

   (a) $y = \displaystyle\int_0^x \sqrt{1+t^2}\,dt$

   (b) $y = \displaystyle\int_0^{\sqrt{x}} \sin(t^2)\,dt$

   (c) $y = \displaystyle\int_0^{\tan x} \frac{1}{1+t^2}\,dt$ (hint: when you simplify your answer, you will get a constant)

3. Find the area of the region enclosed by the curves $y = x^2$ and $y = -x^2 + 4x$.

4. Find the volume of the solid generated by revolving the region bounded by the y-axis and the curve $x = \dfrac{2}{y}$, for $1 \le y \le 4$, about the:

   (a) x-axis, using the Method of Cylindrical Shells

   (b) y-axis

5. Suppose the rate at which the number of people infected with a disease $\dfrac{dy}{dt}$ is proportional to the number of people currently infected y:

   $$\frac{dy}{dt} = ky$$

   Suppose that, in the course of any given year, the number of people infected is reduced by 20%. If there are 10,000 infected people today, how many years will it take to reduce the number to 1000?

6. Find the area of the region enclosed by the curves $x + y^2 = 0$ and $x + 3y^2 = 2$.

7. Find the volume of the following solid:

   The base of the solid is the circle $x^2 + y^2 = 1$. The cross-sections are isosceles right triangles perpendicular to the y-axis.

8. Evaluate the following integrals:

   (a) $\displaystyle\int \frac{x^2}{4-x^2}\,dx$, (b) $\displaystyle\int t\ln t\,dt$, (c) $\displaystyle\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$, (d) $\displaystyle\int x\tan^{-1}x\,dx$,

   (e) $\displaystyle\int \sin^3\theta\,d\theta$, (f) $\displaystyle\int \frac{2}{x^2+2x+5}\,dx$, (g) $\displaystyle\int \frac{\sqrt{1-v^2}}{v^2}\,dv$, (h) $\displaystyle\int \sin\phi\cos^3\phi\,d\phi$

9. In some chemical reactions, the rate at which the amount of a substance changes with time is proportional to the amount present. Consider a substance whose amount obeys the equation:

$$\frac{dy}{dt} = -0.6y$$

   where t is measured in hours. If there are 100 grams of the substance present when $t = 0$, how many grams will be left after 1 hour?

10. Find the volume of a pyramid with height h and base an equilateral triangle with side a.

11. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $y = 2$, and $x = 0$ about the:

    (a) x-axis

    (b) y-axis

12. Evaluate the following integrals:

    (a) $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

    (b) $\displaystyle\int \frac{dx}{\sqrt{1-9x^2}}$

    (c) $\displaystyle\int \tan^{-1}x\,dx$

13. Find the area of the region bounded by the curves $y = e^x$, $y = e$, and $x = 0$.

14. Find the volume of the solid obtained by revolving the region bounded by $y = x^3$, $x = 2$, and $y = 0$ about the y-axis.