

## Fall, 1998 – Final Exam Answers

1. (a)  $y(x) = -4 + \left(\frac{3 - \cos 2x}{2}\right)^2$

(b)  $y(x) = x[c_1 \cos(\sqrt{3} \ln x) + c_2 \sin(\sqrt{3} \ln x)]$

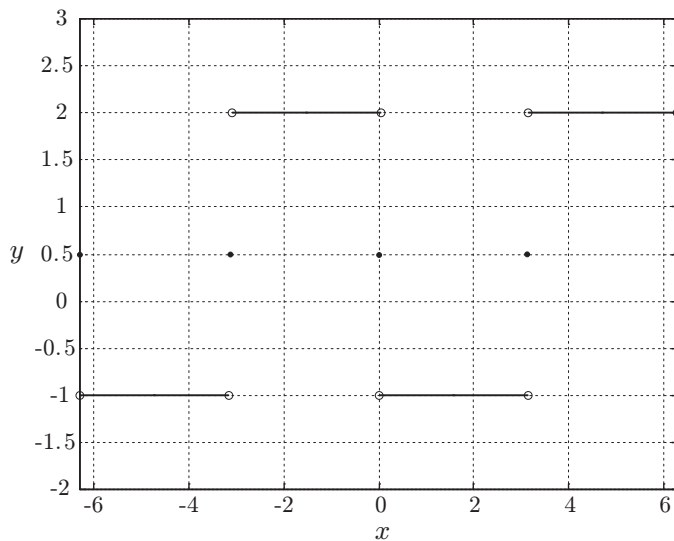
2.  $y(x) = c_1 e^x + c_2 e^{-x} + 2xe^x + 2 \sin 2x$

3. (a)  $u_t = 3u_{xx}$ ,  $u(0, t) = 5$ ,  $u(50, t) = 45$ ,  $u(x, 0) = \begin{cases} 0, & 0 < x < 20 \\ 10, & 20 < x < 50 \end{cases}$

(b)  $v(x) = 5 + \frac{4}{5}x$

(c)  $w_t = 3w_{xx}$ ,  $w(0, t) = 0$ ,  $w(50, t) = 0$ ,  $w(x, 0) = f(x) - v(x)$

4. (a)  $f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{3}{n\pi} [(-1)^n - 1] \sin nx$



(b)

5.  $\lambda = \frac{1}{2}$ ,  $y(x) = Ce^{-x/2} \sin x$

6. (a)  $Y(s) = \frac{2s + 12 + \frac{2}{(s+5)^3} + \frac{s+1}{(s+1)^2+4}}{s^2 + 6s}$

(b)  $\frac{5e^{-3s}}{s} - \frac{5e^{-6s}}{s} - e^{-\pi}$

(c)  $2e^{-2t} \cos 3t + 4e^{-2t} \sin 3t$

7.  $x(t) = \frac{1}{2}e^{t-2} - \frac{1}{2}e^{-(t-2)} - \frac{1}{2}e^t + \frac{1}{2}e^{-t}$ ,  $y(t) = \frac{1}{2}e^{t-2} + \frac{1}{2}e^{-(t-2)} - \frac{1}{2}e^t - \frac{1}{2}e^{-t}$

8. (a)  $t = 450 \text{ min}$

$$(b) \ x(t) = \frac{4t^2 + 400t + 250}{t + 50}$$

$$(c) \ x(t) = 400 - 97.5e^{-t/50}$$

**Bonus:**  $y(x) = c_1x + c_2x^2 - x(\ln x - 1)x + \frac{1}{2}x^2 \ln^2 x$