Math 220 Exam 1 02/23/07

Directions: Answer all questions and show all (intermediate) work in the booklet provided. Start each new question at the top of a new page and $\boxed{\text{box}}$ your final answer.

1. (20 pts) Consider the initial value problem:

$$y' = 2xy(1+x^2)^{-1/2}, \quad y(0) = 1$$

   (a) Is the differential equation separable?

   (b) Is the differential equation linear?

   (c) State the method you will use to solve for $y(x)$ and then find the solution (you may leave your answer in implicit form).

2. (20 pts) Consider the initial value problem: $y' = \dfrac{3}{x} + y$ with $y(1) = -1$.

   (a) Use Euler's method with step size $h = 0.5$ to approximate the solution $y(x)$ at the point $x = 2$.

   (b) Use the improved Euler's method with step size $h = 0.5$ to approximate the solution $y(x)$ at the point $x = 1.5$.

3. (20 pts) Complete each of the following:

   (a) Find the general solution to: $y'' + 4y' + 8y = 0$ (your answer should **not** contain the imaginary number i).

   (b) Write the **form** of the particular solution to: $y'' + 4y' + 8y = 1 + e^{-2x}\cos 2x$ (**do not solve for the coefficients**).

4. (10 pts) A nitric acid solution flows at a constant rate of 4 L/min into a large tank that initially held 100 L of pure water. The solution inside the tank is kept well-stirred and flows out of the tank at a rate of 3 L/min. If the concentration of nitric acid in the solution entering the tank is 0.1, **set up but do not solve** the initial value problem for $x(t)$, the volume of nitric acid in the tank at time t.

5. (30 pts) Consider the following second order, linear, constant coefficient, non-homogeneous differential equation:

$$y'' + 6y' + 5y = 3e^{-2x}.$$

   (a) Use the method of undetermined coefficients to find the particular solution $y_p(x)$ (**you must solve for the coefficient(s)**).

   (b) Find the general solution.

   (c) Now use variation of parameters to find the particular solution $y_p(x)$. Note: If you have done this correctly, you will get the same answer as in part (a).

   The following equations may be helpful:

$$v_1'y_1 + v_2'y_2 = 0$$
$$v_1'y_1' + v_2'y_2' = \frac{g(x)}{a}$$