Math 220 **Exam 2** **04/06/07**

Directions: Answer all questions and show all (intermediate) work in the booklet provided. Start each new question at the top of a new page and box your final answer.

1. (20 pts) Find the general solution to the following system of first order ODEs:

$$\frac{dx}{dt} = x + 3y + 4t$$
$$\frac{dy}{dt} = x - y$$

2. (20 pts) Compute the following expressions:

   (a) $\mathscr{L}\left[t + \sin 3t + e^{2t}\cos t\right]$

   (b) $\mathscr{L}\left[te^{t}\right]$

   (c) $\mathscr{L}^{-1}\left[\dfrac{2}{s(s^2+4)}\right]$

3. (20 pts) Complete each part below:

   (a) Find the function $f(t)$ such that $f(t) = \mathscr{L}^{-1}\left[\dfrac{2e^{-2s} - 4e^{-4s}}{s}\right]$.

   (b) Solve the initial value problem:

$$x'' = f(t), \quad x(0) = 0, \quad x'(0) = 1$$

   where $f(t)$ is the function you found in part (a).

4. (20 pts) Solve the initial value problem:

$$y'' - 4y = 4\delta(t-1), \quad y(0) = 0, \; y'(0) = 0$$

5. (20 pts) Find the general solution for each of the following:

   (a) $x^2y'' - 2y = 0$

   (b) $x^2y'' + 5xy' + 4y = 0$

The following may be useful:

$f(t)$	$F(s) = \mathscr{L}^{-1}[f(t)]$
$f(t)$	$\int_0^\infty e^{-st} f(t)\, dt$
e^{at}	$\dfrac{1}{s-a}$
t^n	$\dfrac{n!}{s^{n+1}}$
$\sin bt$	$\dfrac{b}{s^2+b^2}$
$\cos bt$	$\dfrac{s}{s^2+b^2}$
$u(t-a)$	$\dfrac{e^{-as}}{s}$
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$\delta(t-a)$	e^{-as}
$e^{at}f(t)$	$F(s-a)$
$t^n f(t)$	$(-1)^n \dfrac{d^n}{ds^n} F(s)$
$\int_0^t f(t-v)g(v)\, dv$	$F(s)G(s)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) -$ $\ldots - f^{(n-1)}(0)$