MATH 220 **Combined Final Exam** **6pm-8pm 06 May 1999**

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Directions: Answer **All Questions** and show **All Work** in the **Exam Booklet** provided. Write your **Name**, **Social Security Number**, and **Discussion Section Hour/Day** on the Exam Book Cover Page. Start each new question at the **top** of a new page and **box** your final answer. Each of 8 questions is worth 25 points. **Keep your eyes on your own work and keep your own work covered.** A *Table of Laplace Transforms and a table of Integrals are provided*, but must be returned at the end of the exam. You must also return your **Formulae Sheet** at the end.

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1. Obtain the solution $\mathbf{y(x)}$ to each of the following initial value problems:

   (a) $\mathbf{xy'(x) + 2y(x) = x^2 - x + 1, \quad x > 0, \quad y(1) = \frac{3}{4}}$

   (b) $\mathbf{y'(x) = 2y^2 + xy^2, \quad y(0) = 1}$

2. Find the general solution of

$$\mathbf{x^2 y''(x) - 3xy'(x) + 4y(x) = x^2, \quad x > 0}$$

3. Use the method of undetermined coefficients to find the the general solution of the ODE:

$$\mathbf{y''(x) - 2y'(x) + y(x) = 3 + e^x + 2\sin(x)}$$

4. Find all the values of the real number $\boldsymbol{\lambda > 1}$ for which the boundary value problem

$$\begin{aligned} \textbf{ODE :} \quad & \mathbf{y''(x) + 2y'(x) + \lambda y(x) = 0, \quad 0 \le x \le 1} \\ & \textbf{BC: } \mathbf{y(0) = 0, \quad y(1) = 0} \end{aligned}$$

   has nontrivial solutions, $\mathbf{y(x)}$, and find these solutions.

5. Evaluate:

   (a) $\mathbf{\mathcal{L}\left[e^{-3t}\sin(4t) + (t+2)^2 e^{-4t}\right](s)}$

   (b) $\mathbf{\mathcal{L}^{-1}\left[\frac{s}{(s+2)^2} + \frac{3s+13}{s^2+6s+25}\right](t)}$

6. Find the Laplace Transform of the solutions, $\mathbf{X}(s) = \mathcal{L}[x(t)]$ and $\mathbf{Y}(s) = \mathcal{L}[y(t)]$, satisfying the system of ODEs,

$$x'(t) + 2y(t) = 3e^{t}\cos(t)\delta(t-4), \quad x(0) = 5,$$
$$x(t) - y'(t) = 8u(t-4), \quad y(0) = 0.$$

(Solve, but do not simplify.)

7. Some partial PDE problems:

(a) Apply the method of separation of variables to the PDE ("Wave Equation"),

$$\text{PDE}: \ \frac{\partial^2 u}{\partial t^2}(x,t) = 3\frac{\partial^2 u}{\partial x^2}(x,t), \quad 0 < x < 8, \quad t > 0;$$
$$\text{BC:} \ u(0,t) = 0 = u(8,t), \quad t > 0,$$

and obtain the two ODEs that the function of $\mathbf{x}$ and function of $\mathbf{t}$ must satisfy. Also determine the BC that the function of $\mathbf{x}$ must satisfy (do not solve).

(b) Find the steady state solution only for the inhomogeneous PDE (heat equation) problem,

$$\text{PDE}: \ \frac{\partial u}{\partial t}(x,t) = 3\frac{\partial^2 u}{\partial x^2}(x,t) - 9e^{x}, \quad 0 < x < 8, \quad t > 0;$$
$$\text{BC:} \ u(0,t) = 30, \quad u(8,t) = 70, \quad t > 0,$$

8. (a) Find the Fourier series for the function,

$$f(x) = \begin{Bmatrix} 0, & -\pi < x < -\pi/2 \\ 4, & -\pi/2 < x < +\pi/2 \\ 0, & +\pi/2 < x < +\pi \end{Bmatrix},$$

by computing the Fourier coefficients.

(b) Graph the extended function to which the Fourier series converges in the interval $[-2\pi, +2\pi]$ and be sure to include the values at the points of discontinuity.