Math 220 Quiz 6 Solution

1. Find the general solution to:
$$x^2y'' - 2xy' + 2y = 0$$

2. Classify these functions as being either even, odd, or neither:
   (a) $f(x) = x^2$
   (b) $f(x) = x \ln |x|$
   (c) $f(x) = 2^x$

3. Given that the solution to the heat equation with zero temperature boundary conditions is:
$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2\beta t/L^2} \sin \frac{n\pi x}{L}$$
where $\beta = 1$, $L = \pi$, and the initial condition is:
$$u(x,0) = 2\sin 3x + 5\sin 4x - 6\sin 10x$$
find the coefficients c_n and write the resulting solution $u(x,t)$.

Solution:

1. This is a Cauchy-Euler equation. The *indicial equation* and its roots are:
$$\begin{aligned} r(r-1) - 2r + 2 &= 0 \\ r^2 - 3r + 2 &= 0 \\ (r-2)(r-1) &= 0 \\ \Rightarrow \quad r = 2,\ r &= 1 \end{aligned}$$
The general solution is:
$$\boxed{y(x) = c_1x^2 + c_2x}$$

2. (a) $f(x) = x^2$ is **even** since $f(-x) = (-x)^2 = x^2 = f(x)$
   (b) $f(x) = x\ln|x|$ is **odd** since $f(-x) = (-x)\ln|-x| = -x\ln|x| = -f(x)$
   (c) $f(x) = 2^x$ is **neither**

3. Plugging in $t = 0$, $\beta = 1$, and $L = \pi$ into the solution and setting the result equal to the initial condition we have:
$$u(x,0) = \sum_{n=1}^{\infty} c_n \sin nx = 2\sin 3x + 5\sin 4x - 6\sin 10x$$
All of the c_n will be 0 except for $c_3 = 2$, $c_4 = 5$, and $c_{10} = -6$. Plugging these into the solution we get:
$$\boxed{u(x,t) = 2e^{-9t}\sin 3x + 5e^{-25t}\sin 5x - 6e^{-100t}\sin 10x}$$