Math 220 – Section 1.3 Solutions

1. For the ODE:

$$\frac{dy}{dx} = \frac{4x}{y}$$

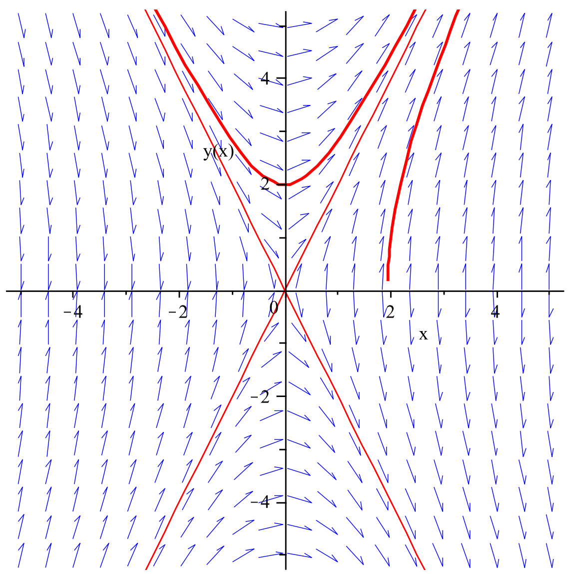
   (a) The lines $y = \pm 2x$ are solutions to the ODE because the equation is satisfied when we plug them into the ODE:

$$\frac{dy}{dx} = \frac{4x}{y}$$
$$\frac{d}{dx}(\pm 2x) = \frac{4x}{\pm 2x}$$
$$\pm 2 = \pm 2$$

   Furthermore, $x \neq 0$. Otherwise, if $x = 0$ then $y = \pm 2(0) = 0$ and $\frac{dy}{dx}$ would be undefined.

   (b) See the figure for the solution satisfying $y(0) = 2$.

   (c) See the figure for the solution satisfying $y(2) = 1$.

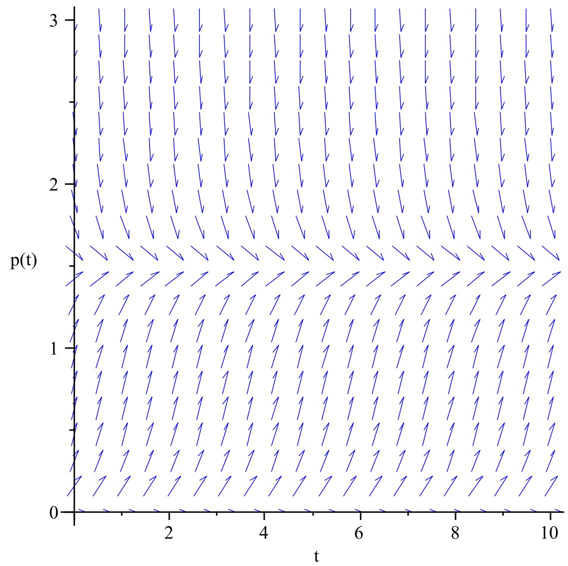


   (d) For part (b), as $x \to +\infty$ the solution tends to infinity and $y = 2x$ is an asymptote. As $x \to -\infty$ the solution tends to infinity and $y = -2x$ is an asymptote. The solution for part (c) tends to infinity and $y = 2x$ is an asymptote. However, there is no solution when $x \to -\infty$ because the solution is not defined for $x < 2$.

5. The logistic equation is given by:

$$\frac{dp}{dt} = 3p - 2p^2$$

   (a) The direction field is shown below:



(b) If $p(0) = 3$, it appears from the direction field that the solution will approach 1.5 as $t \to \infty$.

(c) If $p(0) = 0.8$, it appears from the direction field that the solution will approach 1.5 as $t \to \infty$.

(d) A population of 2000 can never decline to 800. Any solution curve that crosses $p(t) = 2$ will asymptotically approach $p(t) = 1.5$ but will never cross this line.