Math 220 – Section 3.4 Solutions

1. Using $m = 5$ kg, $v_0 = 0$ m/s, and $g = 9.81$ m/s^2, $b = 50$ kg/s, the equation of motion of the object is:

$$\begin{aligned} x(t) &= \frac{mg}{b}t + \frac{m}{b}\left(v_0 - \frac{mg}{b}\right)(1 - e^{-bt/m}) \\ x(t) &= \frac{(5)(9.81)}{50}t + \frac{5}{50}\left(0 - \frac{(5)(9.81)}{50}\right)(1 - e^{-50t/5}) \\ x(t) &= 0.981t - 0.0981(1 - e^{-10t}) \end{aligned}$$

To determine when the ball hits the ground, we must find the value of t such that $x(t) = 1000$. That is, we must solve the equation:

$$0.981t - 0.0981(1 - e^{-10t}) = 1000$$

Assuming that t will be "large", we ignore the exponential term and solve:

$$\begin{aligned} 0.981t - 0.0981(1 - 0) &= 1000 \\ 0.981t &= 1000 + 0.0981 \\ t &= 1019.47 \text{ sec} \end{aligned}$$

2. Using $m = 400/32$ slugs, $v_0 = 0$ ft/s, and $g = 32$ ft/s^2, $b = 10$ slugs/s, the equation of motion of the object is:

$$x(t) = 40t - 50(1 - e^{-0.8t})$$

To determine when the ball hits the ground, we must find the value of t such that $x(t) = 500$. That is, we must solve the equation:

$$40t - 50(1 - e^{-0.8t}) = 500$$

Assuming that t will be "large", we ignore the exponential term and solve:

$$\begin{aligned} 40t - 50(1 - 0) &= 500 \\ 40t &= 500 + 50 \\ t &= 13.75 \text{ sec} \end{aligned}$$

7. During the first part of the motion, we have:

$$m = 75 \text{ kg}, \; v_0 = 0 \text{ m/s}, \; b_1 = 30 \text{ kg/s}$$

Therefore, the velocity and position functions are:

$$\begin{aligned} v(t) &= 24.525 - 24.525e^{-0.4t} \\ x(t) &= 24.525t - 61.3125(1 - e^{-0.4t}) \end{aligned}$$

The time when the velocity is 20 m/s is:

$$\begin{aligned} 24.525 - 24.525e^{-0.4t} &= 20 \\ t &= -\frac{\ln \frac{4.525}{24.525}}{0.4} \\ t &= 4.225 \text{ sec} \end{aligned}$$

At this time, the position is:

$$x(4.225) = 53.62 \text{ m}$$

Once the chute is open, we have:

$$b_2 = 90 \text{ kg/s}, \ v_0 = 20 \text{ m/s}$$

The new equation of motion is:

$$x(t) = \frac{mg}{b_2}t + \frac{m}{b_2}\left(v_0 - \frac{mg}{b_2}\right)(1 - e^{-b_2 t/m})$$
$$x(t) = 8.175t + 9.854(1 - e^{-1.2t})$$

The parachutist has already fallen 53.62 m. The remaining distance to the ground is $2000 - 53.62 = 1946.38$ m. Therefore, to find the time when the parachutist hits the ground we solve:

$$8.175t + 9.854(1 - e^{-1.2t}) = 1946.38$$

Again, assuming that t will be "large", we ignore the exponential term and solve:

$$8.175t + 9.854(1 - 0) = 1946.38$$
$$t = 236.88 \text{ sec}$$

Adding this to the time it took for the velocity to reduce to 20 m/s, the total time is:

$$\text{time} = 4.225 + 236.88 = 241.11 \text{ sec}$$

8. During the first part of the motion, we have:

$$m = 100 \text{ kg}, \ v_0 = 0 \text{ m/s}, \ b_3 = 20 \text{ kg/s}$$

Therefore, the velocity and position functions are:

$$v(t) = 49.05 - 49.05e^{-0.2t}$$
$$x(t) = 49.05t - 245.25(1 - e^{-0.2t})$$

After 30 sec, the velocity and position are:

$$v(30) = 49.05$$
$$x(30) = 1226.25$$

Once the chute is open, we have:

$$b_4 = 100 \text{ kg/s}, \ v_0 = 49.05 \text{ m/s}$$

The new equation of motion is:

$$x(t) = \frac{mg}{b_4}t + \frac{m}{b_4}\left(v_0 - \frac{mg}{b_4}\right)(1 - e^{-b_4 t/m})$$
$$x(t) = 9.81t + 39.24(1 - e^{-t})$$

The parachutist has already fallen 1226.25 m. The remaining distance to the ground is $3000 - 1226.25 = 1773.75$ m. Therefore, to find the time when the parachutist hits the ground we solve:

$$9.81t + 39.24(1 - e^{-t}) = 1773.75$$

Again, assuming that t will be "large", we ignore the exponential term and solve:

$$\begin{aligned} 9.81t + 39.24(1 - 0) &= 1773.25 \\ t &= 176.81 \text{ sec} \end{aligned}$$

Adding this to the 30 seconds of free fall, the total time is:

$$\text{time} = 30 + 176.81 = 196.81 \text{ sec}$$

If the parachutist opens the chute 60 seconds after leaving the helicopter, the total time is:

$$t = 86.81 \text{ sec}$$