Math 220 – Section 4.2 Solutions

1. The general solution to $2y'' + 7y' - 4y = 0$ is
$$y = c_1 e^{t/2} + c_2 e^{-4t}$$

2. The general solution to $y'' + 6y' + 9y = 0$ is
$$y = c_1 e^{-3t} + c_2 t e^{-3t}$$

3. The general solution to $y'' + 5y' + 6y = 0$ is
$$y = c_1 e^{-3t} + c_2 e^{-2t}$$

4. The general solution to $y'' - y' - 2y = 0$ is
$$y = c_1 e^{2t} + c_2 e^{-t}$$

14. The auxiliary equation for $y'' + y' = 0$ is
$$r^2 + r = 0$$
The solutions are $r = 0$ and $r = -1$. Therefore, the general solution and its first derivative are:
$$\begin{aligned} y &= c_1 + c_2 e^{-t} \\ y' &= -c_2 e^{-t} \end{aligned}$$
The initial conditions are $y(0) = 2$, $y'(0) = 1$. The resulting system of equations are:
$$\begin{aligned} c_1 + c_2 &= 2 \\ -c_2 &= 1 \end{aligned}$$
The solution is $c_1 = 3$, $c_2 = -1$. Therefore, the solution is:
$$\boxed{y = 3 - e^{-t}}$$

17. The auxiliary equation for $z'' - 2z' - 2z = 0$ is
$$r^2 - 2r - 2 = 0$$
The solutions are $r = 1 \pm \sqrt{3}$. Therefore, the general solution and its first derivative are:
$$\begin{aligned} z &= c_1 e^{(1+\sqrt{3})t} + c_2 e^{(1-\sqrt{3})t} \\ z' &= (1+\sqrt{3}) c_1 e^{(1+\sqrt{3})t} + (1-\sqrt{3}) c_2 e^{(1-\sqrt{3})t} \end{aligned}$$
The initial conditions are $z(0) = 0$, $z'(0) = 3$. The resulting system of equations are:
$$\begin{aligned} c_1 + c_2 &= 0 \\ (1+\sqrt{3})c_1 + (1-\sqrt{3})c_2 &= 3 \end{aligned}$$
The solution is $c_1 = \frac{\sqrt{3}}{2}$, $c_2 = -\frac{\sqrt{3}}{2}$. Therefore, the solution is:
$$\boxed{y = \frac{\sqrt{3}}{2} e^{(1+\sqrt{3})t} - \frac{\sqrt{3}}{2} e^{(1-\sqrt{3})t}}$$

28. The Wronskian of $y_1(t) = e^{3t}$ and $y_2(t) = e^{-4t}$ is:

$$\begin{aligned} W(y_1, y_2) &= \begin{vmatrix} e^{3t} & e^{-4t} \\ 3e^{3t} & -4e^{-4t} \end{vmatrix} \\ W(y_1, y_2) &= -4e^{-t} - 3e^{-t} \\ W(y_1, y_2) &= -7e^{-t} \end{aligned}$$

There is no value of t on the interval $[0, 1]$ that will make the Wronskian zero. Therefore, y_1 and y_2 are linearly independent.

29. The Wronskian of $y_1(t) = te^{2t}$ and $y_2(t) = e^{2t}$ is:

$$\begin{aligned} W(y_1, y_2) &= \begin{vmatrix} te^{2t} & e^{2t} \\ 2te^{2t} + e^{2t} & 2e^{2t} \end{vmatrix} \\ W(y_1, y_2) &= 2te^{4t} - 2te^{4t} - e^{4t} \\ W(y_1, y_2) &= -e^{4t} \end{aligned}$$

There is no value of t on the interval $[0, 1]$ that will make the Wronskian zero. Therefore, y_1 and y_2 are linearly independent.