Math 220 – Section 4.6 Solutions

1. To find the general solution to
$$y'' + 4y = \tan 2t$$
we note that $y_1 = \sin 2t$ and $y_2 = \cos 2t$ are two solutions to the homogeneous equation. Then, using the variation of parameters equations, we have:
$$v_1'(\sin 2t) + v_2'(\cos 2t) = 0$$
$$v_1'(2\cos 2t) + v_2'(-2\sin 2t) = \tan 2t$$
The solution to the system is:
$$v_1' = \frac{1}{2}\sin 2t$$
$$v_2' = -\frac{1}{2}\frac{\sin^2 2t}{\cos 2t}$$
Integrating, we get:
$$v_1 = -\frac{1}{4}\cos 2t$$
$$v_2 = -\frac{1}{4}\ln|\sec 2t + \tan 2t| + \frac{1}{4}\sin 2t$$
The general solution is:
$$y = c_1y_1 + c_2y_2 + v_1y_1 + v_2y_2$$
$$\boxed{y = c_1\sin 2t + c_2\cos 2t - \frac{1}{4}(\cos 2t)\ln|\sec 2t + \tan 2t|}$$

3. To find the general solution to
$$2x'' - 2x' - 4x = 2e^{3t}$$
we first divide the equation by 2 to get
$$x'' - x' - 2x = e^{3t}$$
Then the homogeneous solutions are $y_1 = e^{2t}$ and $y_2 = e^{-t}$. Using the variation of parameters equations, we have:
$$v_1'(e^{2t}) + v_2'(e^{-t}) = 0$$
$$v_1'(2e^{2t}) + v_2'(-e^{-t}) = e^{3t}$$
The solution to the system is:
$$v_1' = \frac{1}{3}e^{t}$$
$$v_2' = -\frac{1}{3}e^{4t}$$
Integrating, we have:
$$v_1 = \frac{1}{3}e^{t}$$
$$v_2 = -\frac{1}{12}e^{4t}$$

The general solution is:

$$y = c_1 y_1 + c_2 y_2 + v_1 y_1 + v_2 y_2$$

$$\boxed{y = c_1 e^{2t} + c_2 e^{-t} + \frac{1}{4} e^{3t}}$$

4. To find the general solution to

$$y'' - y = 2t + 4$$

we note that $y_1 = e^t$ and $y_2 = e^{-t}$ are two solutions to the homogeneous equation. Then, using the variation of parameters equations, we have:

$$v_1'(e^t) + v_2'(e^{-t}) = 0$$
$$v_1'(e^t) + v_2'(-e^{-t}) = 2t + 4$$

The solution to the system is:

$$v_1' = te^{-t} + 2e^{-t}$$
$$v_2' = -te^t - 2e^t$$

Integrating, we get:

$$v_1 = -e^{-t}(t+1) - 2e^{-t}$$
$$v_2 = -e^t(t-1) - 2e^t$$

The general solution is:

$$y = c_1 y_1 + c_2 y_2 + v_1 y_1 + v_2 y_2$$

$$\boxed{y = c_1 e^t + c_2 e^{-t} - 2t - 4}$$

12. To find the general solution to

$$y'' + y = \tan t + e^{3t} - 1$$

we note that $y_1 = \sin t$ and $y_2 = \cos t$ are two solutions to the homogeneous equation. Then, using the variation of parameters equations, we have:

$$v_1'(\sin t) + v_2'(\cos t) = 0$$
$$v_1'(\cos t) + v_2'(-\sin t) = \tan t + e^{3t} - 1$$

The solution to the system is:

$$v_1' = \sin t + e^{3t}\cos t - \cos t$$
$$v_2' = -\frac{\sin^2 t}{\cos t} - e^{3t}\sin t + \sin t$$

Integrating, we get:

$$v_1 = -\cos t + \frac{3}{10}e^{3t}\cos t + \frac{1}{10}e^{3t}\sin t - \sin t$$
$$v_2 = -\ln|\sec t + \tan t| + \sin t + \frac{1}{10}e^{3t}\cos t - \frac{3}{10}e^{3t}\sin t - \cos t$$

The general solution is:

$$y = c_1 y_1 + c_2 y_2 + v_1 y_1 + v_2 y_2$$

$$\boxed{y = c_1 \sin t + c_2 \cos t - (\cos t)\ln|\sec t + \tan t| + \frac{1}{10}e^{3t} - 1}$$