Math 220 – Section 4.7 Solutions

2. The differential equation
$$t(t-3)y'' + 2ty' - y = t^2$$
written in standard form is
$$y'' + \frac{2}{t-3}y' - \frac{1}{t(t-3)}y = \frac{t}{t-3}$$
We identify the functions p, q, and g as
$$p(t) = \frac{2}{t-3},\ q(t) = \frac{1}{t(t-3)},\ g(t) = \frac{t}{t-3}$$
The functions p and g are continuous everywhere except $t = 3$. The function q is continuous everywhere except $t = 0, 3$. By the existence and uniqueness theorem, we know that there is a unique solution on the interval $(0, 3)$ since the initial conditions are $y(1) = Y_0$ and $y'(1) = Y_1$.

10. The given differential equation is
$$t^2y'' + 2ty' - 6y = 0$$
The characteristic equation and its roots are:
$$\begin{aligned} r(r-1) + 2r - 6 &= 0 \\ r^2 - r + 2r - 6 &= 0 \\ r^2 + r - 6 &= 0 \\ (r+3)(r-2) &= 0 \\ r = -3,\ r &= 2 \end{aligned}$$
The solution is then
$$\boxed{y(t) = c_1t^{-3} + c_2t^2}$$

12. The given differential equation is
$$t^2z'' + 5tz' + 4z = 0$$
The characteristic equation and its roots are:
$$\begin{aligned} r(r-1) + 5r + 4 &= 0 \\ r^2 - r + 5r + 4 &= 0 \\ r^2 + 4r + 4 &= 0 \\ (r+2)^2 &= 0 \\ r &= -2 \end{aligned}$$
The solution is then
$$\boxed{y(t) = c_1t^{-2} + c_2t^{-2}\ln t}$$

15. The given differential equation is

$$y'' - \frac{1}{t}y' + \frac{5}{t^2}y = 0$$

Multiplying the equation by t^2 we get

$$t^2y'' - ty' + 5y = 0$$

The characteristic equation and its roots are:

$$\begin{aligned} r(r-1) - r + 5 &= 0 \\ r^2 - r - r + 5 &= 0 \\ r^2 - 2r + 5 &= 0 \\ r &= -1 \pm 2i \end{aligned}$$

The solution is then

$$\boxed{y(t) = t^{-1}(c_1\cos(2\ln t) + c_2\sin(2\ln t))}$$