Math 220 – Section 4.9 Solutions

3. The solutions to

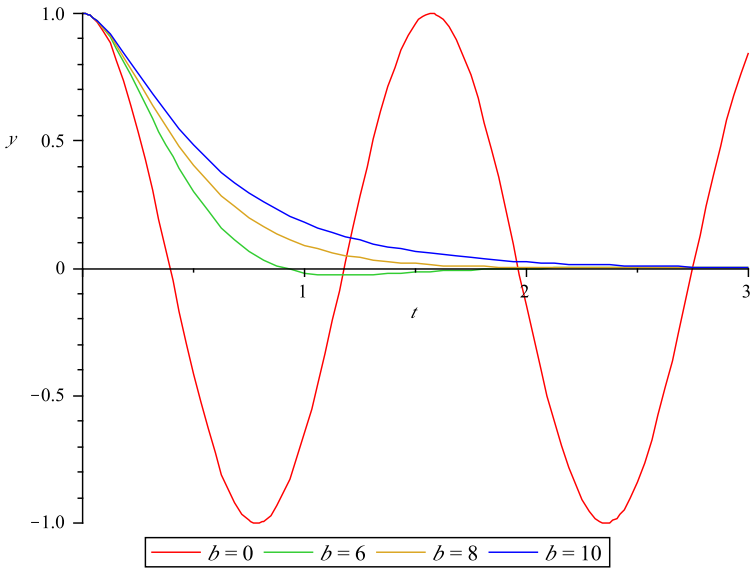
$$y'' + by' + 16y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

for $b = 0,\ 6,\ 8,\ 10$ are:

$$\begin{aligned}
b = 0: &\qquad y(t) = \cos 4t \\
b = 6: &\qquad y(t) = e^{-3t}\left[\frac{3}{\sqrt{7}}\sin\left(\sqrt{7}t\right) + \cos\left(\sqrt{7}t\right)\right] \\
b = 8: &\qquad y(t) = e^{-4t} + 4te^{-4t} \\
b = 10: &\qquad y(t) = \frac{4}{3}e^{-2t} - \frac{1}{3}e^{-8t}
\end{aligned}$$

I plotted the solutions in Maple using the commands:

```
> with(plots):
> plot([cos(4*t),exp(-3*t)*((3/sqrt(7))*sin(sqrt(7)*t)+cos(sqrt(7)*t)),
exp(-4*t)+4*t*exp(-4*t),(4/3)*exp(-2*t)-(1/3)*exp(-8*t)],t=0..3,
legend=['b=0','b=6','b=8','b=10'],labels=['t','y']);
```

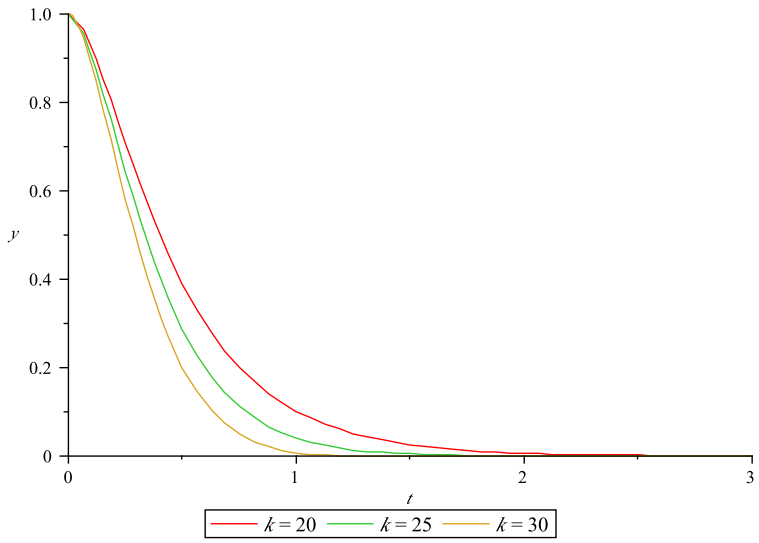


5. The solutions to

$$y'' + 10y' + ky = 0, \quad y(0) = 1, \quad y'(0) = 0$$

for $k = 20,\ 25,\ 30$ are:

$$\begin{aligned}
k = 20: &\qquad y(t) = \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)e^{(-5+\sqrt{5})t} + \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)e^{-(5+\sqrt{5})t} \\
k = 25: &\qquad y(t) = e^{-5t} + 5te^{-5t} \\
k = 30: &\qquad y(t) = \sqrt{5}e^{-5t}\sin\left(\sqrt{5}t\right) + e^{-5t}\cos\left(\sqrt{5}t\right)
\end{aligned}$$



7. When $m = 1/8$ kg, $k = 16$ N/m, $b = 2$ N·s/m, $y(0) = -3/4$ m, and $y'(0) = -2$ m/s the differential equation is:

$$my'' + by' + ky = 0$$
$$\frac{1}{8}y'' + 2y' + 16y = 0$$
$$y'' + 16y' + 128y = 0$$

The solution is:

$$y(t) = -e^{-8t}\sin(8t) - \frac{3}{4}e^{-8t}\cos(8t)$$

The quasiperiod is:

$$\text{quasiperiod} = \frac{2\pi}{\beta} = \frac{2\pi}{8} = \frac{\pi}{4}$$