Math 220 – Section 5.2 Solutions

3. To solve
$$x' + 2y = 0 \tag{1}$$
$$x' - y' = 0 \tag{2}$$
from Equation (2) we have $x' = y'$. Plugging this into Equation (1) we have
$$y' + 2y = 0$$
The solution to this equation is
$$\boxed{y(t) = c_1 e^{-2t}}$$
Since $x' = y'$ we have:
$$\begin{aligned} x' &= y' \\ x' &= -2c_1 e^{-2t} \\ x(t) &= \int -2c_1 e^{-2t}\, dt \\ \Aboxed{x(t) &= c_1 e^{-2t} + c_2} \end{aligned}$$

5. we did this in class

9. To solve
$$x' + y' + 2x = 0 \tag{3}$$
$$x' + y' - x - y = \sin t \tag{4}$$
subtract the two equations to get
$$3x + y = -\sin t \quad \Rightarrow \quad y = -3x - \sin t \tag{5}$$
Plugging this into Equation (3) we have
$$\begin{aligned} x' + y' + 2x &= 0 \\ x' + (-3x - \sin t)' + 2x &= 0 \\ x' - 3x' - \cos t + 2x &= 0 \\ -2x' + 2x &= \cos t \\ x' - x &= -\frac{1}{2}\cos t \end{aligned}$$
The solution to the above equation is
$$\boxed{x(t) = Ce^t + \frac{1}{4}\cos t - \frac{1}{4}\sin t}$$
Then from Equation (5) we have
$$\boxed{y(t) = -3Ce^t - \frac{3}{4}\cos t - \frac{1}{4}\sin t}$$
Note that there is only one unknown constant C because x and y are related by $y = -3x - \sin t$. If we know $x(0)$, then $y(0) = -3x(0) - \sin 0 = -3x(0)$.

19. The initial value problem is

$$\frac{dx}{dt} = 4x + y, \quad x(0) = 1 \tag{6}$$
$$\frac{dy}{dt} = -2x + y, \quad y(0) = 0 \tag{7}$$

From Equation (6) we have

$$y = x' - 4x \tag{8}$$

Plugging this into Equation (7) we have

$$\begin{aligned} y' &= -2x + y \\ (x' - 4x)' &= -2x + (x' - 4x) \\ x'' - 4x' &= -2x + x' - 4x \\ x'' - 5x' + 6x &= 0 \end{aligned}$$

The solution to the above equation is

$$x(t) = c_1 e^{3t} + c_2 e^{2t}$$

Plugging this into Equation (8) we have

$$y(t) = x' - 4x = -c_1 e^{3t} - 2c_2 e^{2t}$$

Using the initial conditions, we get the following system of equations

$$\begin{aligned} x(0) &= c_1 + c_2 = 1 \\ y(0) &= -c_1 - 2c_2 = 0 \end{aligned}$$

The solution to the system is $c_1 = 2$ and $c_2 = -1$. Therefore, the solution to the IVP is

$$\boxed{x(t) = 2e^{3t} - e^{2t}, \quad y(t) = -2e^{3t} + c_2 e^{2t}}$$