Math 220 – Section 5.4 Solutions

7. Find the phase plane equation for the system

$$\frac{dx}{dt} = y - 1 \tag{1}$$
$$\frac{dy}{dt} = e^{x+y} \tag{2}$$

To do this, we will divide Equation (2) by Equation (1) and rearrange terms

$$\begin{aligned} \frac{dy}{dx} &= \frac{e^{x+y}}{y-1} \\ \frac{dy}{dx} &= \frac{e^x e^y}{y-1} \\ \frac{y-1}{e^y}\,dy &= e^x\,dx \\ (ye^{-y} - e^{-y})\,dy &= e^x\,dx \end{aligned}$$

Integrating both sides we get

$$\boxed{-ye^{-y} = e^x + C}$$

11. Solve the phase plane equation for

$$\frac{dx}{dt} = 2y \tag{3}$$
$$\frac{dy}{dt} = 2x \tag{4}$$

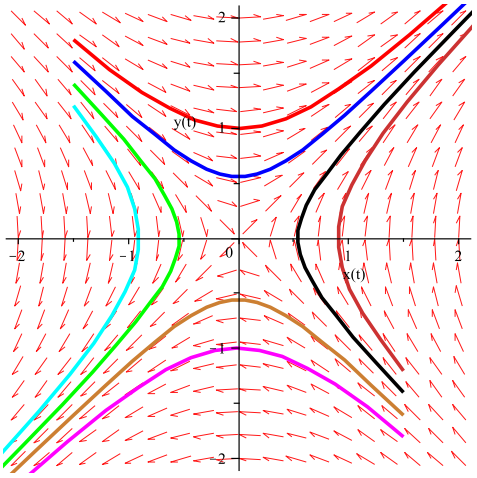
and sketch several representative trajectories. To do this, we will divide Equation (4) by Equation (3) and rearrange terms

$$\begin{aligned} \frac{dy}{dx} &= \frac{2x}{2y} \\ \frac{dy}{dx} &= \frac{x}{y} \\ y\,dy &= x\,dx \end{aligned}$$

Integrating both sides we get

$$\begin{aligned} \frac{1}{2}y^2 &= \frac{1}{2}x^2 + C \\ y^2 - x^2 &= K \end{aligned}$$

A few solutions are plotted below:



13. Solve the phase plane equation for

$$\frac{dx}{dt} = (y-x)(y-1) \tag{5}$$

$$\frac{dy}{dt} = (x-y)(x-1) \tag{6}$$

and sketch several representative trajectories. To do this, we will divide Equation (6) by Equation (5) and rearrange terms

$$\frac{dy}{dx} = \frac{(x-y)(x-1)}{(y-x)(y-1)}$$

$$\frac{dy}{dx} = \frac{-(x-1)}{y-1}$$

$$(y-1)\,dy = -(x-1)\,dx$$

Integrating both sides we get

$$\frac{1}{2}(y-1)^2 = -\frac{1}{2}(x-1)^2 + C$$

$$(x-1)^2 + (y-1)^2 = K$$

A few solutions are plotted below: