Math 220 – Section 7.2 Solutions

3. $\mathscr{L}\{t\} = \dfrac{1}{s^2}$

7. $\mathscr{L}\{e^{2t}\cos 3t\} = \dfrac{s-2}{(s-2)^2+9}$

8. $\mathscr{L}\{e^{-t}\sin 2t\} = \dfrac{2}{(s+1)^2+4}$

9. For

$$f(t) = \begin{cases} 0, & 0 < t < 2 \\ t, & 2 < t \end{cases}$$

we have

$$\begin{aligned}
\mathscr{L}\{f(t)\} &= \int_0^2 e^{-st}\cdot 0\,dt + \int_2^\infty e^{-st}t\,dt \\
&= 0 + \lim_{N\to\infty}\int_2^N e^{-st}t\,dt \\
&= \lim_{N\to\infty}\left[-\frac{1}{s}e^{-st}t - \frac{1}{s^2}e^{-st}\right]_2^N \\
&= \lim_{N\to\infty}\left[-\frac{1}{s}e^{-sN}N - \frac{1}{s^2}e^{-sN} + \frac{2}{s}e^{-2s} + \frac{1}{s^2}e^{-2s}\right] \\
&= e^{-2s}\left(\frac{2}{s}+\frac{1}{s^2}\right)
\end{aligned}$$

12. For

$$f(t) = \begin{cases} e^{2t}, & 0 < t < 3 \\ 1, & 3 < t \end{cases}$$

we have

$$\begin{aligned}
\mathscr{L}\{f(t)\} &= \int_0^3 e^{-st}\cdot e^{2t}\,dt + \int_3^\infty e^{-st}\,dt \\
&= \int_0^3 e^{-(s-2)t}\,dt + \lim_{N\to\infty}\int_3^N e^{-st}\,dt \\
&= \left[-\frac{1}{s-2}e^{-(s-2)t}\right]_0^3 + \lim_{N\to\infty}\left[-\frac{1}{s}e^{-st}\right]_3^N \\
&= -\frac{1}{s-2}e^{-3(s-2)} + \frac{1}{s-2} + \lim_{N\to\infty}\left[-\frac{1}{s}e^{-sN} + \frac{1}{s}e^{-3s}\right] \\
&= -\frac{1}{s-2}e^{-3(s-2)} + \frac{1}{s-2} + \frac{1}{s}e^{-3s}
\end{aligned}$$

13. $\mathscr{L}\{6e^{-3t} - t^2 + 2t - 8\} = \dfrac{6}{s+3} - \dfrac{2}{s^3} + \dfrac{2}{s^2} - \dfrac{8}{s}$

15. $\mathscr{L}\{t^3 - te^t + e^{4t}\cos t\} = \frac{6}{s^4} - \frac{1}{(s-1)^2} + \frac{s-4}{(s-4)^2+1}$

17. $\mathscr{L}\{e^{3t}\sin 6t - t^3 + e^t\} = \frac{6}{(x-3)^2+36} - \frac{6}{s^4} + \frac{1}{s-1}$