Math 220 – Section 7.3 Solutions

3. $\mathscr{L}\{e^{-t}\cos 3t + e^{6t} - 1\} = \dfrac{s+1}{(s+1)^2+9} + \dfrac{1}{s-6} - \dfrac{1}{s}$

5. $\mathscr{L}\{2t^2e^{-t} - t + \cos 4t\} = \dfrac{4}{(s+1)^3} - \dfrac{1}{s^2} + \dfrac{s}{s^2+16}$

7. $\mathscr{L}\{(t-1)^4\} = \mathscr{L}\{t^4 - 4t^3 + 6t^2 - 4t + 1\} = \dfrac{24}{s^5} - \dfrac{24}{s^4} + \dfrac{12}{s^3} - \dfrac{4}{s^2} + \dfrac{1}{s}$

9. $\mathscr{L}\{e^{-t}t\sin 2t\}$. First, let $f(t) = e^{-t}\sin 2t$. Then

$$F(s) = \mathscr{L}\{f(t)\} = \frac{2}{(s+1)^2+4}$$

Since $\mathscr{L}\{tf(t)\} = -\dfrac{dF}{ds}$, we have

$$\mathscr{L}\{e^{-t}t\sin 2t\} = -\frac{d}{ds}\left(\frac{2}{(s+1)^2+4}\right) = \frac{4(s+1)}{[(s+1)^2+4]^2}$$

13. $\mathscr{L}\{\sin^2 t\} = \mathscr{L}\left\{\dfrac{1}{2} - \dfrac{1}{2}\cos 2t\right\} = \dfrac{1}{2s} - \dfrac{s}{2(s^2+4)} = \dfrac{2}{s(s^2+4)}$

25. (a) To find $\mathscr{L}\{t\cos bt\}$, let $f(t) = \cos bt$ and

$$F(s) = \mathscr{L}\{f(t)\} = \frac{s}{s^2+b^2}$$

Then

$$\mathscr{L}\{t\cos bt\} = -\frac{dF}{ds} = -\left[\frac{s^2+b^2-2s^2}{(s^2+b^2)^2}\right] = \frac{s^2-b^2}{(s^2+b^2)^2}$$

(b) To find $\mathscr{L}\{t^2\cos bt\}$, take the second derivative of $F(s)$ in part (a):

$$\begin{aligned}\mathscr{L}\{t^2\cos bt\} = \frac{d^2F}{ds^2} &= -\frac{(s^2+b^2)^2(2s) - 2(s^2-b^2)(s^2+b^2)(2s)}{(s^2+b^2)^4} \\ &= -\frac{2s(s^2+b^2) - 4s(s^2-b^2)}{(s^2+b^2)^3} \\ &= \frac{2s(s^2-3b^2)}{(s^2+b^2)^3}\end{aligned}$$