Math 220 – Section 7.4 Solutions

1. $\mathscr{L}^{-1}\left\{\frac{6}{(s-1)^4}\right\} = e^t t^3$

2. $\mathscr{L}^{-1}\left\{\frac{2}{s^2+4}\right\} = \sin 2t$

3. $\mathscr{L}^{-1}\left\{\frac{s+1}{s^2+2s+10}\right\} = \mathscr{L}^{-1}\left\{\frac{s+1}{(s+1)^2+9}\right\} = e^{-t}\cos 3t$

5. $\mathscr{L}^{-1}\left\{\frac{1}{s^2+4s+8}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{(s+2)^2+4}\right\} = \frac{1}{2}\cdot\frac{2}{(s+2)^2+4} = \frac{1}{2}e^{-2t}\sin 2t$

8. $\mathscr{L}^{-1}\left\{\frac{1}{s^5}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{24}\cdot\frac{24}{s^5}\right\} = \frac{1}{24}t^4$

9.
$$\begin{aligned}
\mathscr{L}^{-1}\left\{\frac{3s-15}{2s^2-4s+10}\right\} &= \mathscr{L}^{-1}\left\{\frac{3}{2}\cdot\frac{s-5}{(s-1)^2+4}\right\} \\
&= \frac{3}{2}\mathscr{L}^{-1}\left\{\frac{s-1-4}{(s-1)^2+4}\right\} \\
&= \frac{3}{2}\mathscr{L}^{-1}\left\{\frac{s-1}{(s-1)^2+4}\right\} - 3\mathscr{L}^{-1}\left\{\frac{2}{(s-1)^2+4}\right\} \\
&= \frac{3}{2}e^t\cos 2t - 3e^t\sin 2t
\end{aligned}$$

21. Performing partial fraction decomposition on $F(s) = \frac{6s^2-13s+2}{s(s-1)(s-6)}$ we have

$$\begin{aligned}
\frac{6s^2-13s+2}{s(s-1)(s-6)} &= \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-6} \\
6s^2-13s+2 &= A(s-1)(s-6) + Bs(s-6) + Cs(s-1)
\end{aligned}$$

Substituting $s = 0, 1, 6$ we get the following solution:

$$\begin{aligned}
s=0: &\quad 2 = 6A \quad \Rightarrow \quad A = \frac{1}{3} \\
s=1: &\quad -5 = -5B \quad \Rightarrow \quad B = 1 \\
s=6: &\quad 140 = 30C \quad \Rightarrow \quad C = \frac{14}{3}
\end{aligned}$$

Therefore,

$$F(s) = \frac{1}{3}\cdot\frac{1}{s} + \frac{1}{s-1} + \frac{14}{3}\cdot\frac{1}{s-6}$$

The inverse Laplace transform is

$$\mathscr{L}^{-1}\{F(s)\} = \frac{1}{3} + e^t + \frac{14}{3}e^{6t}$$

23. Performing partial fraction decomposition on $F(s) = \dfrac{5s^2+34s+53}{(s+3)^2(s+1)}$ we have

$$\begin{aligned}\frac{5s^2+34s+53}{(s+3)^2(s+1)} &= \frac{A}{s+3} + \frac{B}{(s+3)^2} + \frac{C}{s+1}\\ 5s^2+34s+53 &= A(s+3)(s+1) + B(s+1) + C(s+3)^2\end{aligned}$$

Substituting $s = -1, -3, 0$ we get the following solutions:

$$\begin{aligned} s=-1: &\quad 24 = 4C \quad \Rightarrow \quad C = 6\\ s=-3: &\quad -4 = -2B \quad \Rightarrow \quad B = 2\\ s=0: &\quad 53 = 3A + B + 9C \quad \Rightarrow \quad A = -1\end{aligned}$$

Therefore,

$$F(s) = -\frac{1}{s+3} + \frac{2}{(s+3)^2} + \frac{6}{s+1}$$

The inverse Laplace transform is

$$\mathscr{L}^{-1}\{F(s)\} = -e^{-3t} + 2e^{-3t}t + 6e^{-t}$$

25. Performing partial fraction decomposition on $F(s) = \dfrac{7s^2+23s+30}{(s-2)(s^2+2s+5)}$ we have

$$\begin{aligned}\frac{7s^2+23s+30}{(s-2)(s^2+2s+5)} &= \frac{A}{s-2} + \frac{B(s+1)+C}{(s+1)^2+4}\\ 7s^2+23s+30 &= A[(s+1)^2+4] + [B(s+1)+C](s-2)\end{aligned}$$

Substituting $s = 2, -1, 0$ we get the following solutions:

$$\begin{aligned} s=2: &\quad 104 = 13A \quad \Rightarrow \quad A = 8\\ s=-1: &\quad 14 = 4A - 3C \quad \Rightarrow \quad C = 6\\ s=0: &\quad 30 = 5A - 2B - 2C \quad \Rightarrow \quad B = -1\end{aligned}$$

Therefore,

$$F(s) = \frac{8}{s-2} + \frac{-(s+1)+6}{(s+1)^2+4}$$

The inverse Laplace transform is

$$\mathscr{L}^{-1}\{F(s)\} = 8e^{2t} - e^{-t}\cos 2t + 3e^{-t}\sin 2t$$