# Math 220 – Section 7.6 Solutions

1. $\mathscr{L}\left\{(t-1)^2u(t-1)\right\} = e^{-s}\mathscr{L}\left\{t^2\right\} = \boxed{\dfrac{2}{s^3}e^{-s}}$

5. The function

$$g(t) = \begin{cases} 0, & 0 < t < 1 \\ 2, & 1 < t < 2 \\ 1, & 2 < t < 3 \\ 3, & 3 < t \end{cases}$$

can be written as follows:

$$\boxed{g(t) = 2u(t-1) - u(t-2) + 2u(t-3)}$$

Its Laplace Transform is

$$\boxed{\mathscr{L}\left\{g(t)\right\} = \frac{2e^{-s}}{s} - \frac{e^{-2s}}{s} + \frac{2e^{-3s}}{s}}$$

7. From the graph we have

$$\boxed{g(t) = tu(t-1) + (1-t)u(t-2)}$$

Its Laplace Transform is

$$\boxed{\mathscr{L}\left\{g(t)\right\} = \left(\frac{2}{s^2} + \frac{1}{s}\right)e^{-s} + \left(-\frac{2}{s^2} - \frac{1}{s}\right)e^{-2s}}$$

15. We use partial fractions to rewrite the given function as follows:

$$\begin{aligned} F(s) &= \frac{se^{-3s}}{s^2+4s+5} \\ &= \frac{s}{(s+2)^2+1}e^{-3s} \\ &= \frac{s+2}{(s+2)^2+1}e^{-3s} - 2\frac{1}{(s+2)^2+1}e^{-3s} \end{aligned}$$

Its inverse Laplace Transform is

$$\boxed{f(t) = \mathscr{L}^{-1}\left\{F(s)\right\} = e^{-2(t-3)}\cos(t-3)u(t-3) - 2e^{-2(t-3)}\sin(t-3)}$$

17. We rewrite the given function as follows:

$$F(s) = \frac{e^{-3s}(s-5)}{(s+1)(s+2)} = \left(\frac{7}{s+2} - \frac{6}{s+1}\right)e^{-3s}$$

Its inverse Laplace Transform is

$$\boxed{f(t) = \mathscr{L}^{-1}\left\{F(s)\right\} = 7e^{-2(t-3)} - 6e^{-(t-3)}u(t-3)}$$

25. From the given graph, we write $f(t)$ as follows:

$$\begin{aligned} f(t) &= 1 - u(t-a) + u(t-2a) - u(t-3a) + \dots \\ &= 1 + \sum_{n=1}^{\infty} (-1)^n u(t-na) \end{aligned}$$

The Laplace transform of $f$ is

$$\begin{aligned} \mathscr{L}\{f\} &= \frac{1}{s} - \frac{e^{-as}}{s} + \frac{e^{-2as}}{s} - \frac{e^{-3as}}{s} + \dots \\ &= \frac{1}{s}\left(1 - e^{-as} + e^{-2as} - e^{-3as} + \dots\right) \\ &= \frac{1}{s}\left[1 + \left(-e^{-as}\right) + \left(-e^{-as}\right)^2 + \left(-e^{-as}\right)^3 + \dots\right] \\ &= \frac{1}{s}\left[\frac{1}{1-(-e^{-as})}\right] \\ &= \boxed{\frac{1}{s(1+e^{-as})}} \end{aligned}$$

In the above analysis, we used the fact that the expression in brackets is a geometric series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

where $x = -e^{-as}$.