Math 220 – Section 7.7 Solutions

1.

$$\begin{aligned}
y'' - 2y' + y &= g(t) \\
\mathscr{L}[y''] - 2\mathscr{L}[y'] + \mathscr{L}[y] &= \mathscr{L}[g(t)] \\
s^2Y(s) - sy(0) - y'(0) - 2(sY(s) - y(0)) + Y(s) &= G(s) \\
s^2Y(s) + s - 1 - 2sY(s) - 2 + Y(s) &= G(s) \\
Y(s)(s^2 - 2s + 1) &= G(s) - s + 3 \\
Y(s) &= \frac{G(s)}{s^2 - 2s + 1} - \frac{s-3}{s^2 + 2s + 1} \\
\Rightarrow \quad y(t) &= \mathscr{L}^{-1}\left[\frac{1}{(s-1)^2}G(s) - \frac{s-1}{(s-1)^2} + \frac{2}{(s-1)^2}\right] \\
y(t) &= te^t * g(t) - e^t + 2te^t \\
\boxed{y(t) &= (2t-1)e^t + \int_0^t (t-v)e^{t-v}g(v)\,dv}
\end{aligned}$$

4.

$$\begin{aligned}
y'' + y &= g(t) \\
\mathscr{L}[y''] + \mathscr{L}[y] &= \mathscr{L}[g(t)] \\
s^2Y(s) - sy(0) - y'(0) + Y(s) &= G(s) \\
s^2Y(s) - 1 + Y(s) &= G(s) \\
Y(s)(s^2 + 1) &= G(s) + 1 \\
Y(s) &= \frac{G(s)}{s^2+1} + \frac{1}{s^2+1} \\
\Rightarrow \quad y(t) &= \mathscr{L}^{-1}\left[\frac{1}{s^2+1}G(s) + \frac{1}{s^2+1}\right] \\
y(t) &= \sin t * g(t) + \sin t \\
\boxed{y(t) &= \sin t + \int_0^t \sin(t-v)g(v)\,dv}
\end{aligned}$$

5.

$$\begin{aligned}
\mathscr{L}^{-1}\left[\frac{1}{s(s^2+1)}\right] &= \mathscr{L}^{-1}\left[\frac{1}{s}\frac{1}{s^2+1}\right] \\
&= 1 * \sin t \\
&= \int_0^t \sin v\,dv \\
&= [-\cos v]_0^t \\
&= \boxed{1 - \cos t}
\end{aligned}$$