Math 220 – Section 8.5 Solutions

2. To find the general solution to $2x^2y'' + 13xy' + 15y = 0$ we solve for the roots of the indicial equation:

$$\begin{aligned} 2r(r-1) + 13r + 15 &= 0 \\ 2r^2 + 11r + 15 &= 0 \\ (2r+5)(r+3) &= 0 \\ \Rightarrow \quad r = -\frac{5}{2},\ r &= -3 \end{aligned}$$

The roots are real and distinct. Therefore, the general solution is:

$$\boxed{y(x) = c_1x^{-5/2} + c_2x^{-3}}$$

4. To find the general solution to $x^2y'' + 2xy' - 3y = 0$ we solve for the roots of the indicial equation:

$$\begin{aligned} r(r-1) + 2r - 3 &= 0 \\ r^2 + r - 3 &= 0 \\ r &= \frac{-1 \pm \sqrt{1^2 - 4(-3)(1)}}{2} \\ r &= \frac{-1 \pm \sqrt{13}}{2} \end{aligned}$$

The roots are real and distinct. Therefore, the general solution is:

$$\boxed{y(x) = c_1x^{(-1+\sqrt{13})/2} + c_2x^{(-1-\sqrt{13})/2}}$$

5. Rewriting the ODE we have:

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{5}{x}\frac{dy}{dx} - \frac{13}{x^2}y \\ x^2y'' - 5xy' + 13y &= 0 \end{aligned}$$

The roots of the indicial equation are:

$$\begin{aligned} r(r-1) - 5r + 13 &= 0 \\ r^2 - 6r + 13 &= 0 \\ r &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(13)(1)}}{2} \\ r &= \frac{6 \pm \sqrt{-16}}{2} \\ r &= 3 \pm 2i \end{aligned}$$

The roots are complex with $\alpha = 3$ and $\beta = 2$. Therefore, the general solution is:

$$\boxed{y(x) = x^3\left[c_1\cos(2\ln x) + c_2\sin(2\ln x)\right]}$$

15. The initial value problem is:

$$t^2x'' - 12x = 0,\ x(1) = 3,\ x'(1) = 5$$

The indicial equation and its roots are:

$$
\begin{aligned}
r(r-1) - 12 &= 0 \\
r^2 - r - 12 &= 0 \\
(r-4)(r+3) &= 0 \\
\Rightarrow \quad r = 4,\ r &= -3
\end{aligned}
$$

The roots are real and distinct. Therefore, the general solution is:

$$x(t) = c_1 t^4 + c_2 t^{-3}$$

Before we plug in the initial conditions, we must compute x':

$$x'(t) = 4c_1 t^3 - 3c_2 t^{-4}$$

Plugging in the initial conditions, we get the system of equations:

$$
\begin{aligned}
x(1) = c_1 + c_2 &= 3 \\
x'(1) = 4c_1 - 3c_2 &= 5
\end{aligned}
$$

The solution to this system is $c_1 = 2$ and $c_2 = 1$. Therefore, the solution is:

$$\boxed{x(t) = 2t^4 + t^{-3}}$$