

Math 417, Complex Analysis, Final Exam
Thursday, August 8, 2008

YOU MUST SHOW ALL OF YOUR COMPUTATIONS IN THE EXAM BOOKLET TO RECEIVE FULL CREDIT

1. (30 pts) Determine all possible series representations of the function:

$$f(z) = \frac{1}{z(z^2 + 1)}$$

about $z = 0$ and state their regions of validity.

2. (20 pts) Find all points at which

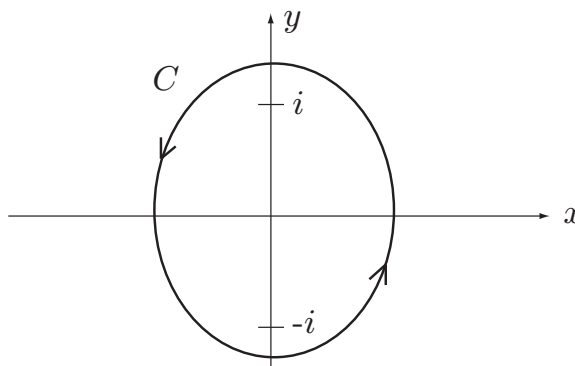
$$f(z) = \frac{x}{x^2 + y^2} - i \frac{y}{x^2 + y^2}$$

is differentiable. At what points is f analytic? Explain.

3. (40 pts) Compute each of the following integrals:

(a) $\int_C \sin\left(\frac{1}{z}\right) dz$ where C is the circle $|z| = 1$ oriented counterclockwise.

(b) $\int_C \frac{e^z}{z(z^2 + 1)} dz$ where the contour C is shown below:



4. (30 pts) Compute the improper integral:

$$I = \int_0^{\infty} \frac{dx}{x^2 + x + 1}$$

by considering the integral:

$$\int_C \frac{\log z}{z^2 + z + 1} dz$$

where C is the contour depicted in Figure 99 on p. 274.

5. (30 pts) Complete each of the following:

(a) Find all values of z^π where $z = 2 + 2i$.

(b) Determine the principal value of $\log z$ where $z = -1 - i$.

(c) How many solutions of $3e^z - z = 0$ are in the disk $|z| \leq 1$? Explain.

6. (25 pts) Evaluate the improper integral:

$$\int_0^\infty \frac{x \sin ax}{x^2 + b^2} dx$$