MATH 417 **Homework 2**
Instructor: D. Cabrera **Due June 23**

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1. Find and sketch the image of the rectangle $0 < x < 1$, $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$ under the transformation $w = e^z$.

2. Sketch the following sets and determine whether they are open, closed, or neither.

   (a) $|z+3| < 2$

   (b) $|\operatorname{Im} z| < 1$

   (c) $0 < |z-1| < 2$

   (d) $\operatorname{Re} z = 1$

   (e) $|z-4| \geq |z|$

3. Show that

   (a) $\displaystyle\lim_{z\to 3} \frac{2}{z-3} = \infty$

   (b) $\displaystyle\lim_{z\to\infty} \frac{z^2+1}{3z^2-4} = \frac{1}{3}$

4. Find $f'(z)$ for the following functions:

   (a) $f(z) = 4z^2 + 5z - 3$

   (b) $f(z) = (2-z^3)^2$

   (c) $f(z) = \dfrac{z+2}{3z-2}$ where $z \neq \dfrac{2}{3}$

5. Show that $f'(z)$ does not exist at any point z when $f(z) = \operatorname{Re} z$.

6. Let $z = x + iy$. Determine the values of z for which the Cauchy-Riemann equations are satisfied for the following functions:

   (a) $f(z) = e^{-x}e^{-iy}$

   (b) $f(z) = 2x + ixy^2$

   (c) $f(z) = x^2 + iy^2$

   (d) $f(z) = \operatorname{Im} z$