MATH 417 **Homework 3**
Instructor: D. Cabrera **Due June 30**

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1. Let a function $f(z) = u + iv$ be differentiable at z_0.

   (a) Use the Chain Rule and the formulas $x = r\cos\theta$ and $y = r\sin\theta$ to show that

   $$u_x = u_r \cos\theta - u_\theta \frac{\sin\theta}{r}, \quad v_x = v_r \cos\theta - v_\theta \frac{\sin\theta}{r}$$

   (b) Then use the Cauchy-Riemann equations in polar coordinates

   $$ru_r = v_\theta, \quad u_\theta = -rv_r$$

   and the fact that $f'(z_0) = u_x + iv_x$ to show that

   $$f'(z_0) = e^{-i\theta}(u_r + iv_r)$$

2. Show that the function $f(z) = e^{-y}\sin x - ie^{-y}\cos x$ is entire.

3. Show that the function $f(z) = xy + iy$ is not analytic at any point in the complex plane.

4. Let $u(x, y) = \dfrac{y}{x^2 + y^2}$.

   (a) Show that $u(x, y)$ is harmonic in the domain D which is the set of all points z in the complex plane excluding $z = 0$.

   (b) Find the most general harmonic conjugate v of u.

5. Find all values of each expression.

   (a) $\exp\left(2 - \frac{\pi}{4}i\right)$

   (b) $\log(-2 + 2i)$

   (c) $\operatorname{Log}(ei)$

6. Show that the function $f(z) = e^{2z}$ is entire and write an expression for $f'(z)$ in terms of z.

7. Show that $\operatorname{Log}(-1 + i)^2 \neq 2\operatorname{Log}(-1 + i)$.