MATH 417 **Homework 4**
Instructor: D. Cabrera **Due July 7**

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1. Find all values of each expression below.
   (a) $(1-i)^i$
   (b) $\cos(1-i)$
   (c) $\sin^{-1}(1-i)$

2. Prove that $\sin(2z) = 2\sin z\cos z$ by using the definitions of $\sin z$ and $\cos z$.

3. Find the values of z for which $\cos z = 0$ by using the fact that
$$|\cos z|^2 = \cos^2 x + \sinh^2 y \qquad \text{where} \qquad \sinh y = \frac{e^y - e^{-y}}{2}$$

4. Show that $f(z) = \sin(\bar{z})$ is analytic nowhere.

5. Evaluate the integral
$$\int_C e^z\,dz$$
where C is the contour consisting of the two straight-line segments: (1) from $z = i$ to $z = 1+i$ and (2) from $z = 1+i$ to $z = 1-2i$.

6. Evaluate the integral
$$\int_C (z^2-1)\,dz$$
where C is the semicircle $z = e^{it}$, $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ oriented counterclockwise.

7. Show that
$$\left|\int_C \frac{2z+1}{z^2-4}\,dz\right| \leq \pi$$
where C is the upper half of the circle $|z| = 1$ oriented counterclockwise. Justify your answer.

8. Find an upper bound on
$$\left|\int_C \frac{dz}{z^2+1}\right|$$
where C is the circle $|z-i| = 1$ oriented counterclockwise. Justify your answer.