

1. Find all singular points of the given function. For each isolated singular point, classify the point as being a removable singularity, a pole of order N (specify N), or an essential singularity.

(a) $f(z) = \frac{1}{z(z-1)}$

(b) $f(z) = \frac{e^z - 1}{z^3}$

(c) $f(z) = \sin\left(\frac{1}{z}\right)$

2. Find all residues of $f(z) = \frac{1}{(z+4)(z-1)^3}$.

3. Evaluate $\int_C \frac{(z+1)^2}{z^2(z-1)} dz$ where C is the circle $|z| = 3$ oriented counterclockwise.

4. Evaluate $\int_C \frac{e^z}{\sin z} dz$ where C is the circle $|z - \pi| = 1$ oriented counterclockwise.

5. Show that $\int_0^\infty \frac{dx}{x^4 + 1} = \frac{\pi}{2\sqrt{2}}$.

6. Show that $\int_{-\infty}^\infty \frac{dx}{(x^2 + 4)^3} = \frac{3\pi}{256}$