Math 417, Complex Analysis, Midterm Exam
Friday, July 11, 2008

YOU MUST SHOW ALL OF YOUR COMPUTATIONS IN THE EXAM BOOKLET TO RECEIVE FULL CREDIT

1. Find all values of:

   (a) $\log(3-4i)$ (b) $(2+2i)^i$

2. Complete each of the following:

   (a) Is $|e^z| = e^{|z|}$? Explain.

   (b) Explain why the following reasoning is incorrect:

   $$|e^{iz}| = |\cos z + i\sin z| = \sqrt{\cos^2 z + \sin^2 z} = 1 \quad \text{for all } z$$

3. Determine the values of z for which the function $f(z) = xe^z$ is analytic. If f is analytic at $z = 0$, then compute $f'(0)$.

4. Consider the function $u(x,y) = e^{2x}\sin(2y) + 2x$.

   (a) Show that $u(x,y)$ is harmonic in the entire z plane.

   (b) Find a harmonic conjugate $v(x,y)$ of $u(x,y)$. Then express $f = u + iv$ as a function of z.

5. Let C be a contour consisting of the two straight-line segments: (1) from $z = i$ to $z = 1+i$ and (2) from $z = 1+i$ to $z = 1-2i$. Compute the integral:

   $$I = \int_C e^z\,dz$$

   (a) by finding a parametric representation $z(t) = x(t) + iy(t)$, $a \le t \le b$ for each line segment and computing:

   $$\int_a^b f(z(t))z'(t)\,dt$$

   over each arc of the contour and

   (b) verifying the result above by using an antiderivative $F(z)$ of $f(z) = e^z$.

6. Consider the integral:

$$I = \int_C \frac{dz}{z(z+5)}$$

where C is the rectangle with corners at $z = 3+3i$, $z = -3+3i$, $z = -3-3i$, and $z = 3-3i$, oriented counterclockwise.

(a) Find an upper bound on $|I|$. Justify your answer.

(b) Compute the exact value of $|I|$.