

Math 417 – Handout Solutions

Using the Ratio Test on each series, we have:

$$1. \lim_{n \rightarrow \infty} \left| \frac{(n+1)z^{n+1}}{nz^n} \right| = |z| \lim_{n \rightarrow \infty} \frac{n+1}{n} = |z|.$$

In order for the series to converge, we need $|z| < 1$. Therefore, the radius of convergence is 1.

$$2. \lim_{n \rightarrow \infty} \left| \frac{z^{n+1}}{n+1} \cdot \frac{n}{z^n} \right| = |z| \lim_{n \rightarrow \infty} \frac{n}{n+1} = |z|.$$

In order for the series to converge, we need $|z| < 1$. Therefore, the radius of convergence is 1.

$$3. \lim_{n \rightarrow \infty} \left| \frac{z^{n+1}}{(n+1)^{n+1}} \cdot \frac{n^n}{z^n} \right| = |z| \lim_{n \rightarrow \infty} \frac{1}{n+1} \cdot \left(\frac{n}{n+1} \right)^n = |z| \cdot 0 \cdot e^{-1} = 0.$$

Since the limit is less than 1 for all z , the radius of convergence is ∞ .