

Math 417 – Section 11 Solutions

1. (a) The domain of definition of $f(z) = \frac{1}{z^2 + 1}$ is all z except $z = \pm i$.
(d) The domain of definition of $f(z) = \frac{1}{1 - |z|^2}$ is all z except for all points on the circle $|z| = 1$.
2. Taking the function $f(z) = z^3 + z + 1$ and letting $z = x + iy$ we have:

$$\begin{aligned} f(z) &= (x + iy)^3 + (x + iy) + 1 \\ &= x^3 + 3ix^2y + 3i^2xy^2 + i^3y^3 + x + iy + 1 \\ &= x^3 + 3ix^2y - 3xy^2 - iy^3 + x + iy + 1 \\ &= (x^3 - 3xy^2 + x + 1) + i(3x^2y - y^3 + y) \end{aligned}$$