

Math 417 – Section 12 Solutions

1. Letting $z = x + iy$ in the transformation $w = z^2$ we have:

$$w = (x^2 - y^2) + i(2xy)$$

So we have:

$$u = x^2 - y^2$$

$$v = 2xy$$

The curves in the z -plane that correspond to the curves $u = 1$, $u = 2$, $v = 1$, and $v = 2$ are:

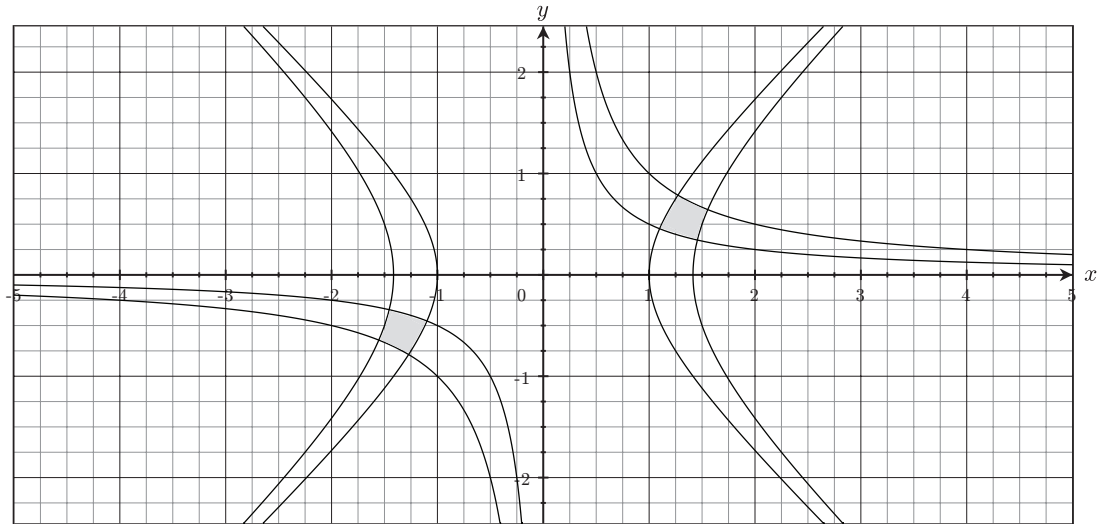
$$u = 1 : x^2 - y^2 = 1$$

$$u = 2 : x^2 - y^2 = 2$$

$$v = 1 : 2xy = 1$$

$$v = 2 : 2xy = 2$$

Plotting these in the z -plane, we have:



Either of the shaded regions is a domain whose image is the given rectangle in the w -plane.

6. In Figure 7, Appendix 2, the region in the z -plane is bounded by $y = 0$, $y = \pi$, and $x = 0$. The image of this region under the transformation $w = e^z$ can be found by determining the images of the boundaries. First, from the transformation we have:

$$u = e^x \cos y$$

$$v = e^x \sin y$$

The curve $x = 0$, $0 \leq y \leq \pi$ is represented parametrically by:

$$u = \cos y$$

$$v = \sin y$$

which is the upper half of the circle of radius 1 centered at the origin in the w -plane. The curve $y = 0$, $x \leq 0$ is represented parametrically by:

$$\begin{aligned}u &= e^x \\v &= 0\end{aligned}$$

which is the line $v = 0$, $0 \leq u \leq 1$ in the w -plane. The curve $y = \pi$, $x \leq 0$ is represented parametrically by:

$$\begin{aligned}u &= -e^x \\v &= 0\end{aligned}$$

which is the line $v = 0$, $-1 \leq u \leq 0$ in the w -plane. Therefore, the image is the region inside and on the boundary of the upper half of the circle of radius 1 centered at the origin in the w -plane.