

Math 417 – Sections 18 & 19 Solutions

1. (b) The derivative of $f(z) = (1 - 4z^2)^3$ is:

$$f'(z) = 3(1 - 4z^2)^2 \frac{d}{dz}(1 - 4z^2) = -24z(1 - 4z^2)^2$$

- (c) The derivative of $f(z) = \frac{z-1}{2z+1}$ ($z \neq -\frac{1}{2}$) is:

$$f'(z) = \frac{(2z+1)(1) - (z-1)(2)}{(2z+1)^2} = \frac{2}{(2z+1)^2}$$

8. (b) Show that $f'(z)$ does not exist at any point z when $f(z) = \operatorname{Re} z$.

Let $z = x + iy$. Then $f(z) = \operatorname{Re} z = \operatorname{Re}(x + iy) = x + i0$. The quotient $\Delta w / \Delta z$ is:

$$\frac{\Delta w}{\Delta z} = \frac{f(z + \Delta z) - f(z)}{\Delta z} = \frac{x + \Delta x - x}{\Delta x + i\Delta y} = \frac{\Delta x}{\Delta x + i\Delta y}$$

If we take $\Delta x = 0$ then the quotient is:

$$\frac{\Delta w}{\Delta z} = \frac{0}{0 + i\Delta y} = 0$$

If we take $\Delta y = 0$ then the quotient is:

$$\frac{\Delta w}{\Delta z} = \frac{\Delta x}{\Delta x + i0} = 1$$

Since we get two different values of the quotient along two different paths to z , $f'(z)$ does not exist at any point z .

9. First, we write an expression for $\Delta w / \Delta z$ at $z = 0$:

$$\begin{aligned} \frac{\Delta w}{\Delta z} &= \frac{f(0 + \Delta z) - f(0)}{\Delta z} \\ &= \frac{f(\Delta z) - 0}{\Delta z} \\ &= \frac{(\overline{\Delta z})^2 / \Delta z}{\Delta z} \\ &= \frac{(\overline{\Delta z})^2}{(\Delta z)^2} \\ &= \frac{(\Delta x - i\Delta y)^2}{(\Delta x + i\Delta y)^2} \end{aligned}$$

On the real axis, we have $\Delta y = 0$ so the quotient becomes:

$$\frac{\Delta w}{\Delta z} = \frac{(\Delta x - i0)^2}{(\Delta x + i0)^2} = \frac{(\Delta x)^2}{(\Delta x)^2} = 1$$

On the imaginary axis, we have $\Delta x = 0$ so the quotient becomes:

$$\frac{\Delta w}{\Delta z} = \frac{(0 - i\Delta y)^2}{(0 + i\Delta y)^2} = \frac{(-i)^2(\Delta y)^2}{i^2(\Delta y)^2} = 1$$

Along the line where $\Delta y = \Delta x$, the quotient becomes:

$$\begin{aligned}\frac{\Delta w}{\Delta z} &= \frac{(\Delta x - i\Delta x)^2}{(\Delta x + i\Delta x)^2} \\ &= \frac{(\Delta x)^2(1-i)^2}{(\Delta x)^2(1+i)^2} \\ &= \frac{(1-i)^2}{(1+i)^2} \\ &= \frac{1-2i+i^2}{1+2i+i^2} \\ &= \frac{-2i}{2i} \\ &= -1\end{aligned}$$

Since the quotient is different along two different paths, $f'(0)$ does not exist.