

Math 417 – Sections 1 & 2 Solutions

1. (a) $(\sqrt{2} - i) - i(1 - \sqrt{2}i) = \sqrt{2} - i - i + \sqrt{2}i^2 = \sqrt{2} - 2i - \sqrt{2} = -2i$
 (b) Using the product definition $(x_1, y_1)(x_2, y_2) = (x_1x_2 - y_1y_2, y_1x_2 + x_1y_2)$, we have:

$$(2, -3)(-2, 1) = ((2)(-2) - (-3)(1), (-2)(-3) + (2)(1)) \\ = (-1, 8)$$

- (c) First, we have:

$$(3, 1)(3, -1) = ((3)(3) - (1)(-1), (3)(1) + (3)(-1)) \\ = (10, 0)$$

Then, we have:

$$(3, 1)(3, -1) \left(\frac{1}{5}, \frac{1}{10} \right) = (10, 0) \left(\frac{1}{5}, \frac{1}{10} \right) \\ = \left((10) \left(\frac{1}{5} \right) - (0) \left(\frac{1}{10} \right), \left(\frac{1}{5} \right) (0) + (10) \left(\frac{1}{10} \right) \right) \\ = (2, 1)$$

2. Let $z = x + iy$. Then,

$$(a) \operatorname{Re}(iz) = \operatorname{Re}(i(x + iy)) = \operatorname{Re}(ix + i^2y) = \operatorname{Re}(-y + ix) = -y = -\operatorname{Im}(z)$$

$$(b) \operatorname{Im}(iz) = \operatorname{Im}(i(x + iy)) = \operatorname{Im}(ix + i^2y) = \operatorname{Im}(-y + ix) = x = \operatorname{Re}(z)$$

5. Let $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. Then,

$$z_1 z_2 = (x_1, y_1)(x_2, y_2) \\ = (x_1x_2 - y_1y_2, y_1x_2 + x_1y_2) \\ = (x_2x_1 - y_2y_1, y_2x_1 + x_2y_1) \\ = (x_2, y_2)(x_1, y_1) \\ = z_2 z_1$$

10. To solve $z^2 + z + 1 = 0$, we let $z = (x, y)$:

$$z^2 + z + 1 = 0 \\ (x, y)(x, y) + (x, y) + (1, 0) = (0, 0) \\ (x^2 - y^2, 2xy) + (x, y) + (1, 0) = (0, 0) \\ (x^2 - y^2 + x + 1, 2xy + y) = (0, 0)$$

Therefore, x and y must satisfy the equations:

$$x^2 - y^2 + x + 1 = 0 \tag{1}$$

$$2xy + y = 0 \tag{2}$$

Note that $y = 0$ is a solution to Equation (2). That would mean that $z = (x, 0)$ is a real number. However, $z^2 + z + 1 = 0$ does not have a real solution (using the quadratic equation would result in a negative discriminant). Therefore, $y \neq 0$ and Equation (2) then gives us:

$$x = -\frac{1}{2}$$

Plugging into Equation (1) and solving for y gives us:

$$\begin{aligned}x^2 - y^2 + x + 1 &= 0 \\ \left(-\frac{1}{2}\right)^2 - y^2 - \frac{1}{2} + 1 &= 0 \\ \frac{1}{4} - y^2 + \frac{1}{2} &= 0 \\ y^2 &= \frac{3}{4} \\ y &= \pm \frac{\sqrt{3}}{2}\end{aligned}$$

Thus, the solutions are:

$$z = \left(-\frac{1}{2}, \pm \frac{\sqrt{3}}{2}\right)$$