

Math 417 – Section 31 Solutions

1. If $\operatorname{Re} z_1 > 0$ and $\operatorname{Re} z_2 > 0$ then we know that:

$$-\frac{\pi}{2} < \Theta_1 < \frac{\pi}{2}, \quad -\frac{\pi}{2} < \Theta_2 < \frac{\pi}{2}$$

where Θ_1 and Θ_2 are the principal arguments of z_1 and z_2 , respectively. Thus, we have:

$$-\pi < \Theta_1 + \Theta_2 < \pi$$

Letting $z_1 = r_1 \exp(i\Theta_1)$ and $z_2 = r_2 \exp(i\Theta_2)$ we have:

$$z_1 z_2 = r_1 r_2 \exp(i(\Theta_1 + \Theta_2))$$

Since $-\pi < \Theta_1 + \Theta_2 < \pi$, the principal argument of $z_1 z_2$ is $\Theta_1 + \Theta_2$.

The principal logarithm of $z_1 z_2$ is then:

$$\begin{aligned} \operatorname{Log}(z_1 z_2) &= \ln(r_1 r_2) + i(\Theta_1 + \Theta_2) \\ &= \ln r_1 + \ln r_2 + i\Theta_1 + i\Theta_2 \\ &= (\ln r_1 + i\Theta_1) + (\ln r_2 + i\Theta_2) \\ &= \operatorname{Log}(z_1) + \operatorname{Log}(z_2) \end{aligned}$$